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FIRST-DERIVATIVE FEEDBACK COMPENSATION
OF CONTROL SYSTEMS

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COMPENSATION OF CONTROL SYSTEMS

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Donald E. Kirk

FIRST-DERIVATIVE FEEDBACK
COMPENSATION OF CONTROL SYSTEMS

by

Donald E. Kirk
//

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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COMPENSATION OF CONTROL SYSTEMS

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ABSTRACT

This investigation is a study of feedback compensation of feedback control systems. The problem is restricted to encompass only compensation utilizing first-derivative or velocity feedback.

An unique block-diagram reduction is developed and used throughout the investigation in conjunction with root-locus plots for the system studied.

Qualitative and quantitative illustrations are developed to emphasize the salient features of the technique.

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1. Introduction

This investigation was carried out in order to determine the usefulness of an unique approach to the analysis and design of feedback control systems. Specifically, feedback compensation of second and higher-order systems was attempted.

The technique employed was to perform block-diagram manipulations which allowed the effects of a given compensation device in a feedback path to be studied as if the compensator were cascaded in a forward path in the system. Root locus methods were used throughout the study. To illustrate the technique utilized, consider a general block diagram for a feedback control system. In Figure 1, a typical control system is illustrated. An arbitrary compensation device, $G_c(s)$, has been introduced in a minor feedback path. Two possible feedback paths are shown. Consider the case where $G_c(s)$ is shown in a minor loop around $G_2(s)$. Using standard block-diagram reduction, the following result for the system transfer function is obtained:

$$\frac{\theta_c}{\theta_R}(s) = \frac{G_1(s)G_2(s)}{1 + G_2(s)G_c(s) + G_1(s)G_2(s)} \quad (1-1)$$

The denominator of the system function determines the characteristic equation for the system. $G_2(s)G_c(s) + G_1(s)G_2(s)$ may be expressed as the ratio of two polynomials in s when $G_1(s)$ and $G_2(s)$ are known and $G_c(s)$ has been selected. Suppose, however, it is deemed desirable to be able to express the characteristic equation in the form:

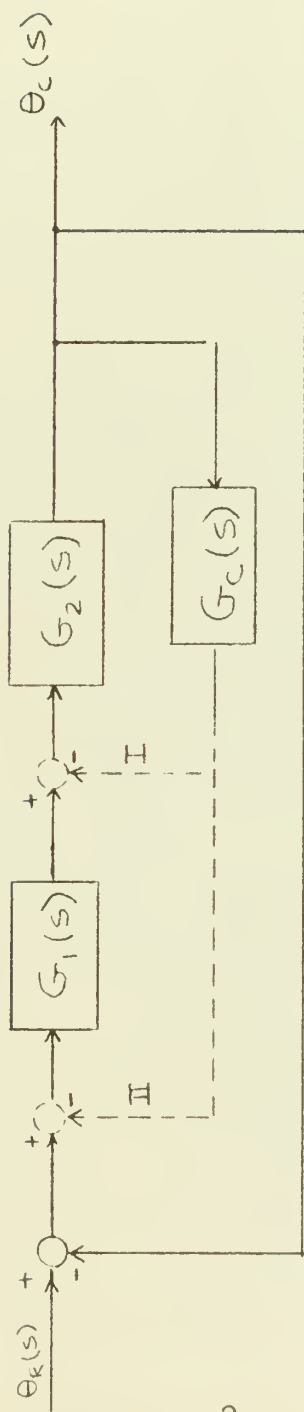


FIGURE 1 GENERALIZED BLOCK DIAGRAM FOR A CONTROL SYSTEM

$$1 + G_c(s) \left[\frac{P(s)}{Q(s)} \right] = 0$$

(1-2)

where $P(s)$ and $Q(s)$ represent polynomials in s . This form allows a more easily obtainable evaluation of the merits of various selections for $G_c(s)$.

The characteristic equation of the form of equation (1-2) might be obtained by selecting a specific $G_c(s)$ and by algebraic manipulations arriving at the desired form. This means, however, that these manipulations must be re-performed for each different choice of $G_c(s)$. Since our interest is focused entirely upon the characteristic equation, it is of no interest what sort of a transfer function is obtained, as long as the characteristic equation obtained by block-diagram manipulations is the same as that obtained in the standard manner.

Again consider Figure 1 with $G_c(s)$ still in the minor loop around $G_2(s)$. The block diagram may be rearranged as shown in Figure 2. The only restriction which must be observed is that the direction of signal flow through the blocks remains as it was prior to rearrangement. Notice in Figure 2 that the revised system function will be $\theta_c(s)/B(s)$ which is not the same as $\theta_c(s)/\theta_r(s)$. Then:

$$\frac{\theta_c}{B}(s) = \frac{\frac{G_2(s)}{1 + G_1(s) G_2(s)}}{1 + \frac{G_2(s) G_c(s)}{1 + G_1(s) G_2(s)}} \quad (1-3)$$

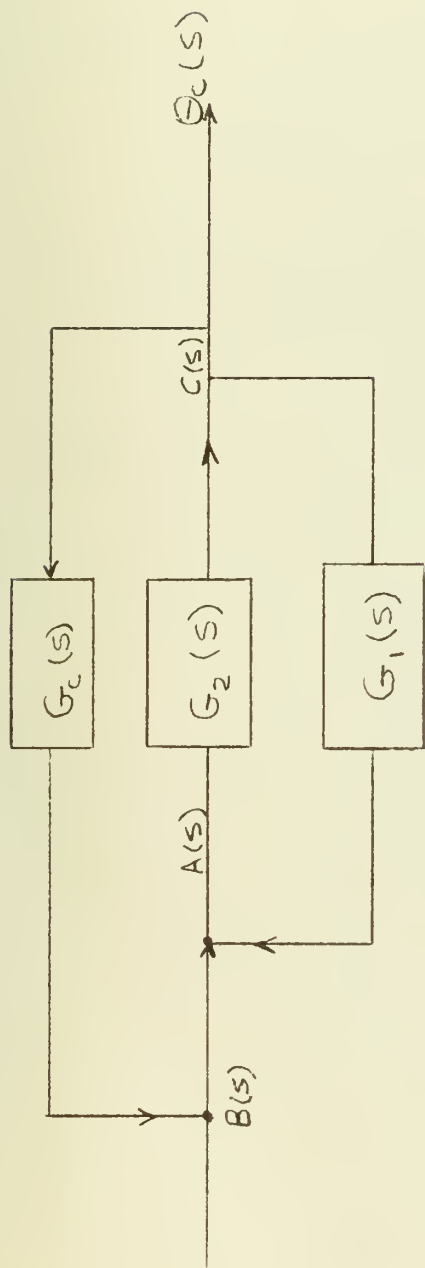


FIGURE 2 MODIFIED BLOCK DIAGRAM FOR THE SYSTEM IN FIGURE 1

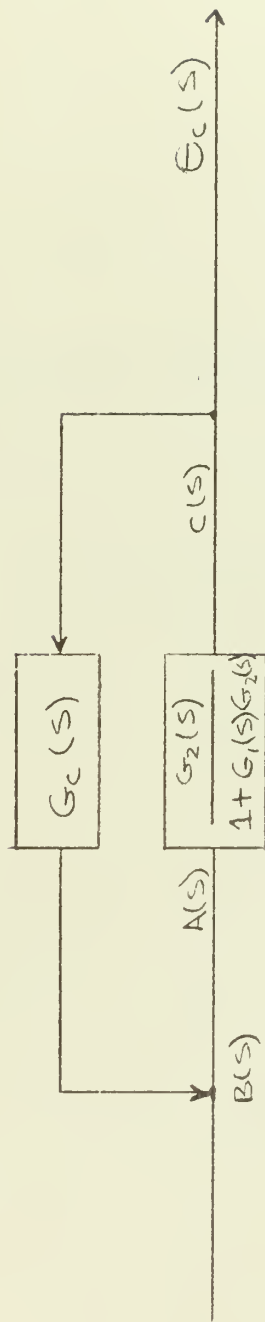


FIGURE 3 BLOCK DIAGRAM AFTER REDUCTION

Therefore, the characteristic equation is:

$$1 + \frac{G_2(s) G_c(s)}{1 + G_1(s) G_2(s)} = 0 \quad (1-3)$$

or,

$$1 + G_c(s) \left[\frac{P(s)}{Q(s)} \right] = 0 \quad (1-5)$$

where:

$$P(s) = G_2(s)$$

$$Q(s) = 1 + G_1(s) G_2(s)$$

The next question is then: are the two characteristic equations identical?

To answer this question, refer again to equation (1-1).

$$\frac{\theta_c}{\theta_R}(s) = \frac{G_1(s) G_2(s)}{1 + G_2(s) G_c(s) + G_1(s) G_2(s)} \quad (1-1)$$

If both numerator and denominator are divided by $1 + G_1(s) G_2(s)$:

$$\frac{\theta_c}{\theta_R}(s) = \frac{G_1(s) G_2(s)}{1 + G_1(s) G_2(s)} \div \frac{1 + G_c(s) \left[\frac{G_2(s)}{1 + G_1(s) G_2(s)} \right]}{1 + G_1(s) G_2(s)} \quad (1-6)$$

2. A General Statement of the Compensation Problem

The roots of the characteristic equation describe the response of a given system to various types of forcing functions. For a second-order system, the location of the roots in the s domain is readily correlated with system performance in the time domain. A summary of the criteria for evaluating the performance of a second-order system should clearly define the problem:

1. Damping ratio, ζ ; a measure of the transient overshoot.
2. Settling time, $5/\zeta\omega_N$; the time required for a system to complete its transient.
3. The minimum allowable forward gain to overcome coulomb friction, stiction and to give the desired output for a small input signal.
4. The minimum allowable forward gain to prevent error due to a load disturbance.
5. Bandwidth; the range of frequencies within which a signal will be attenuated no more than a certain amount. This specification is usually given to insure minimization of the effect of noise interference.
6. The transient oscillating frequency.
7. The error coefficient, K_p , K_v , or K_a depending on the type number of the system.

Criteria such as the damping ratio, settling time, the natural oscillating angular velocity, ω_N , and the transient oscillating angular velocity, ω_t , may be found by inspection of the root locations for a second-order system. For higher-order systems, these quantities are not as simply defined; however, if the system has one pair of roots which dominate the system response, the locations of these roots may serve as an approximate indication of the system behavior in the time domain.

Notice that the denominator obtained for $\Theta_c(s)/\Theta_r(s)$ and $\Theta_c(s)/B(s)$ are identical; however, the numerators are not the same. Reduction by standard block-diagram methods and subsequent manipulations of the resulting characteristic equation become more difficult as the system becomes more complex.

This study is confined to the study of first-derivative feedback as a possible means of compensating a system. A quantitative investigation of a second-order system is included, and a qualitative examination of higher-order systems is also contained.

3. Second-order Systems

Consider the system shown in Figure 1. Let:

$$G_1(s) = K_1 \quad (\text{an ideal amplifier})$$

and $G_2(s) = K_2 / s(s + P)$ (an ideal d-c motor) P is the location of the motor pole. $G_c(s) = K_t s$

Equation (1-4) gives the characteristic equation as:

$$1 + G_c(s) \frac{G_2(s)}{1 + G_1(s) G_2(s)} = 0 \quad (1-4)$$

$$1 + K_t s \frac{\frac{K_2}{s(s+P)}}{1 + \frac{K_1 K_2}{s(s+P)}} = 0 \quad (1-4a)$$

$$1 + K_t s \frac{K_2}{s(s+P) + K_1 K_2} = 0 \quad (1-4b)$$

$$K_t s \frac{K_2}{s(s+P) + K_1 K_2} = -1 \quad (1-4c)$$

The root locus for the uncompensated system is shown in Figure 4. The root locus for the compensated system is shown in Figure 5. The location of the poles will depend on the magnitude of P , K_1 , and K_2 . If the poles are complex conjugates as shown, the root locus is a circular arc, centered

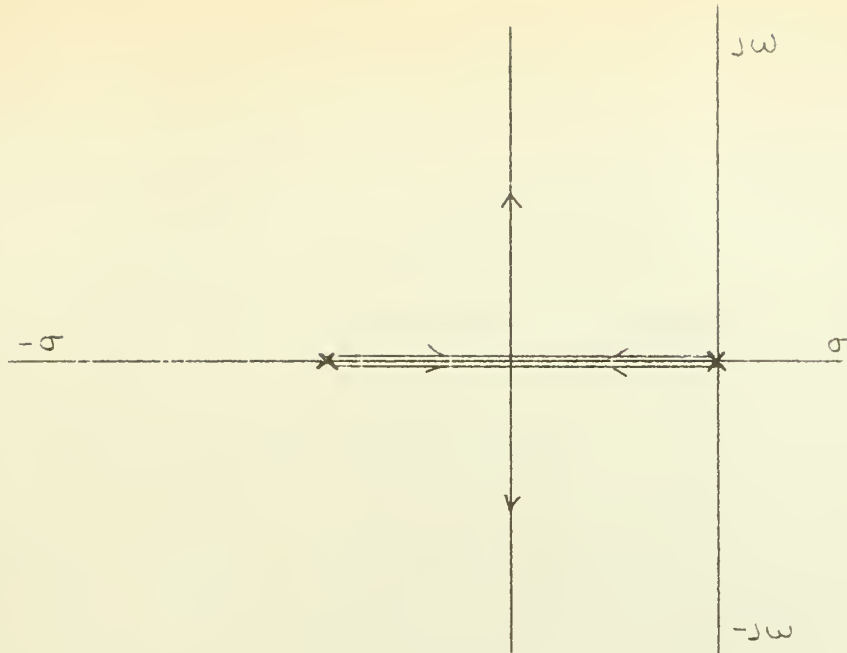


FIGURE 4 ROOT LOCUS FOR THE UNCOMPENSATED SYSTEM

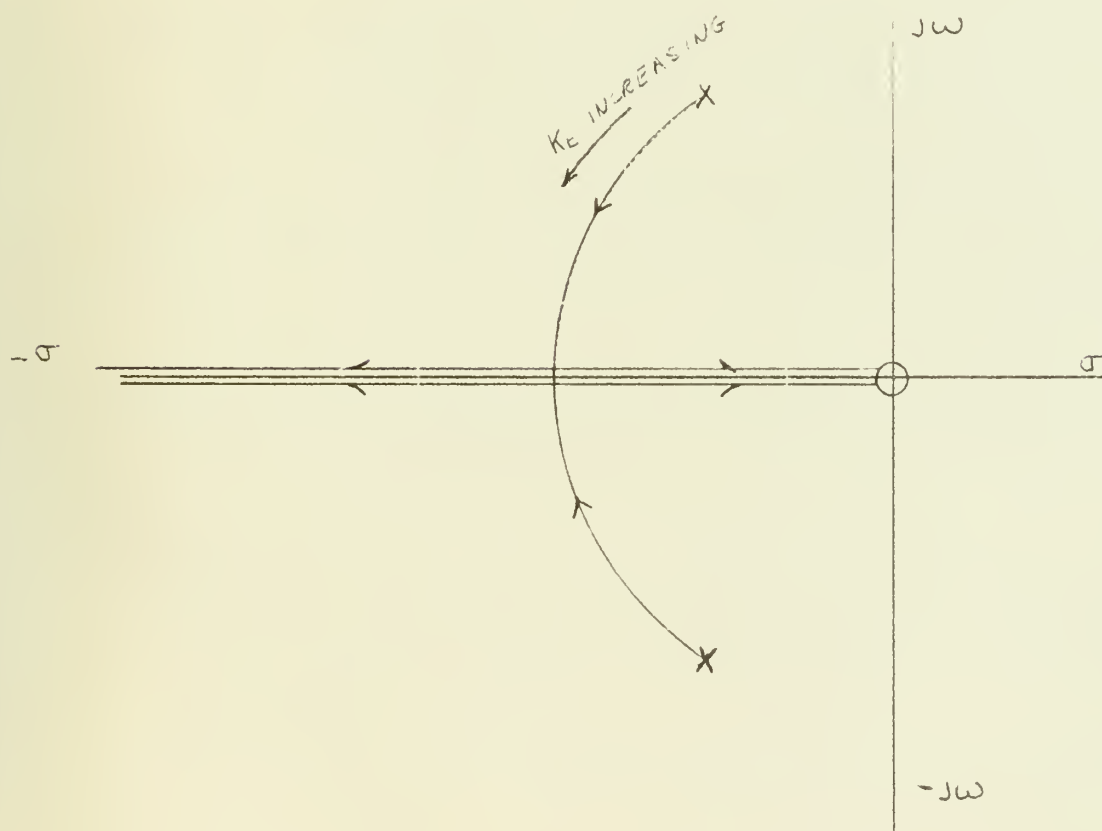


FIGURE 5 ROOT LOCUS FOR THE COMPENSATED SYSTEM

at the origin, with a radius equal to ω_N . The circular-arc shape of the root locus and the radius equal to ω_N suggests that the root locus may be normalized by dividing both coordinate scales by the factor $\omega_N = \sqrt{K_1 K_2}$. This actually means the performance of a coordinate transformation in which:

$$w = f(s) = \frac{s}{\omega_N} \quad (3-1)$$

where: $s = \sigma + j\omega$ and $w = a + jb$

It may be shown that the resulting root locus is a circular arc with a radius of 1. (See Figure 6)

It may be derived that (see Appendix A1) the location of the complex conjugate poles, P' and P'^* , may be expressed as:

$$P' = -\frac{P}{2\omega_N} + j \sqrt{1 - \frac{P^2}{4\omega_N^2}} \quad (3-2)$$

$$P'^* = -\frac{P}{2\omega_N} - j \sqrt{1 - \frac{P^2}{4\omega_N^2}}$$

In general, the product $K_1 K_2$ will be specified by threshold requirements of the system. The design problem, then, may be of one or two types:

1. Find the value of K_t which gives roots at the desired locations.
2. Given a maximum value for K_t , find K_1 and K_2 to give roots at the desired locations.

If the root locus is plotted and normalized, it may be shown (see Appendix A2) that:

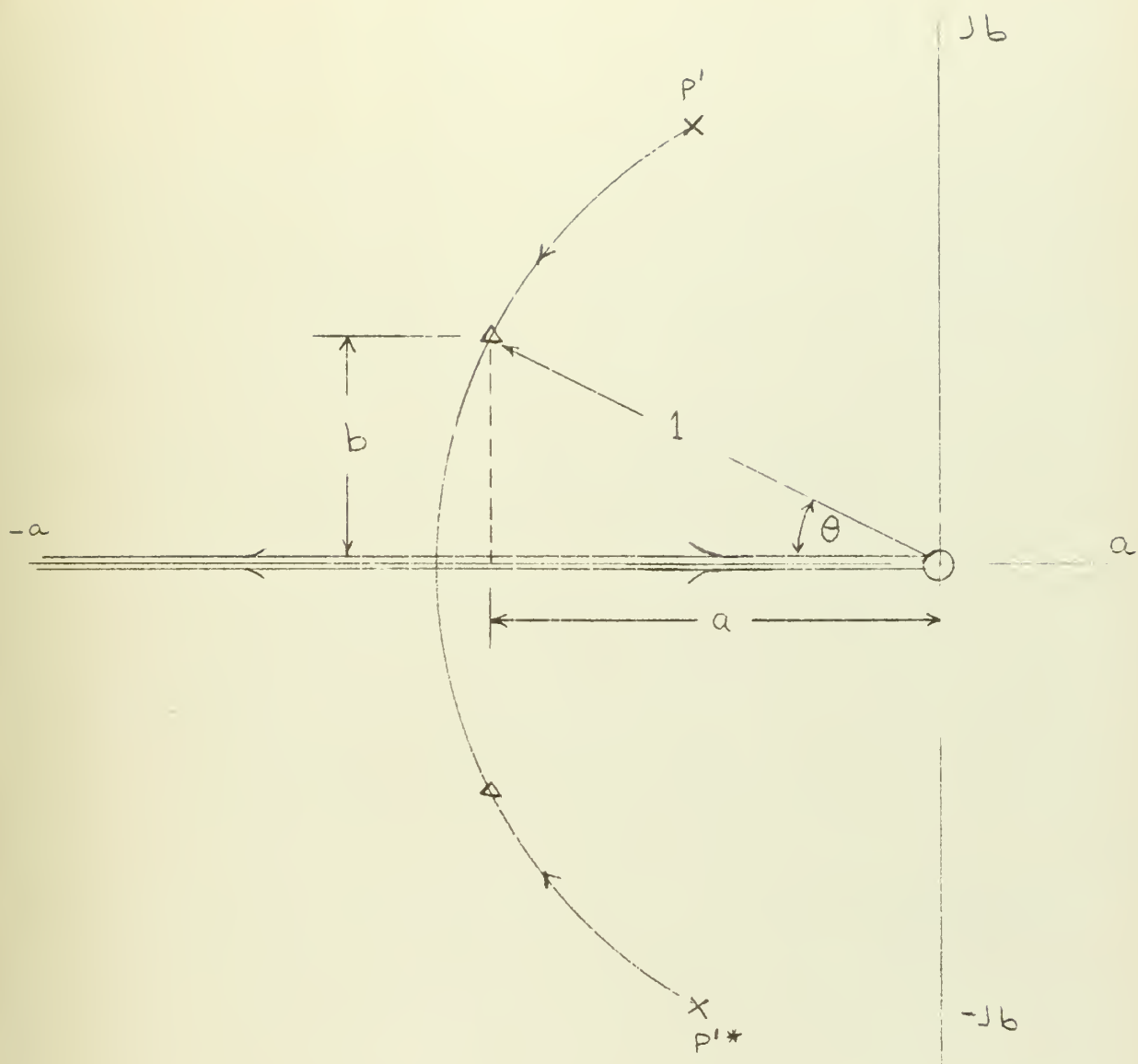


FIGURE 6 NORMALIZED ROOT LOCUS FOR THE COMPENSATED SYSTEM

$$K_t = \frac{2a\omega_N - P}{K_2} \quad (3-3)$$

where a is the magnitude of the real part of the desired root.

Therefore, in order to find K_t , the magnitude of a is found and the above relationship may be used. The procedure may be simplified, however, if it is noted in Figure 6 that:

$$\zeta = \cos \theta = \cos \left[\cos^{-1} \frac{a}{1} \right] \quad (3-4)$$

$$\zeta = a \quad (3-4a)$$

Therefore:

$$K_t = \frac{2\zeta\omega_N - P}{K_2} \quad (3-5)$$

It may be shown (see Appendix A3) that this result can be obtained directly from the expression for the damping ratio derived from the system block diagram. Therefore, there is no necessity for even drawing the root locus; equation (3-5) may be used directly. The technique is illustrated, however, as a possible method for more complex systems.

From equation (1-4b) it is seen that the characteristic equation is:

$$S(S+P) + K_1 K_2 + K_2 K_t S = 0 \quad (3-6)$$

$$S^2 + (P + K_2 K_t)S + K_1 K_2 = 0 \quad (3-6a)$$

For a second-order system, the characteristic equation is expressed in general terms as:

$$S^2 + 2\zeta\omega_N S + \omega_N^2 = 0 \quad (3-7)$$

Therefore:

$$\omega_N = \sqrt{K_1 K_2} \quad (3-8)$$

$$\zeta = \frac{P + K_2 K_t}{2 \sqrt{K_1 K_2}} \quad (3-9)$$

For this system, it may also be shown (see Appendix A4) that:

$$\begin{aligned} K_V &= \lim_{S \rightarrow 0} S \left[\frac{K_1 K_2}{S(S + P + K_2 K_t)} \right] \\ &= \frac{K_1 K_2}{P + K_2 K_t} \end{aligned} \quad (3-10)$$

or

$$K_V = \frac{\omega_N}{2\zeta} \quad (3-11)$$

Equation (3-11) reveals a most useful relationship which occurs in a second-order system of this type. The information contained in equation (3-11) may be illustrated on a graph which may be used as a nomogram. The nomogram is shown in Figure 7.

Thus a plot of K_v as a function of ω_N for different values of ζ can be made. Therefore, for a system which is of the form of Figure 1, the feasibility of utilizing tachometer feedback compensation may be determined simply by expressing the desired specifications in terms of K_v , ω_N , and ζ , and inspecting the nomogram shown in Figure 7. The utility of this nomogram will be illustrated in the numerical examples included in this investigation.

The technique to be utilized may be summarized as follows:

1. The system specifications are used to define root locations which are suitable.
2. The nomogram shown in Figure 7 is used to determine whether or not the system may be compensated satisfactorily using tachometer feedback.
3. Equation (3-5) is solved to find the variable quantity of interest.

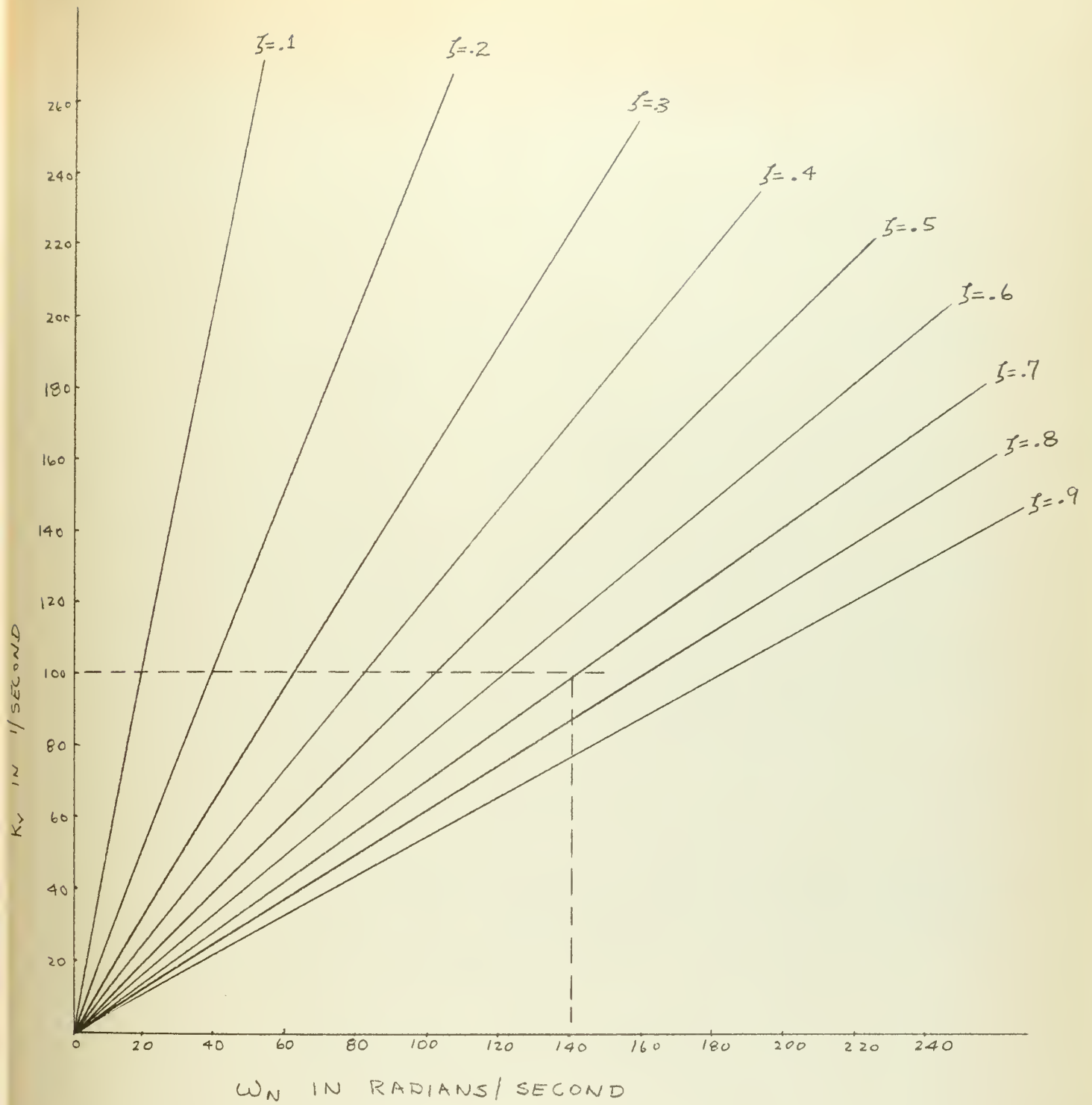


FIGURE 7 NOMOGRAM FOR SECOND-ORDER SYSTEM

4. Quantitative Investigation of a Second-order System

In this section, some numerical examples are developed to illustrate the procedure. These examples may be divided into two categories:

1. Steady-state specifications for allowable error to a step-position input.
2. Steady-state specifications for allowable error to a step-position input, and a ramp-position input.

For the system in question, it will be specified that there is to be no steady-state position error to a step-position input; that is, the system must be a type-1 system.

It should be noted that for examples of classification 1, the nomogram shown in Figure 7 is unnecessary due to the fact that it will always be possible to compensate the system because of the lack of a requirement concerning steady-state-position error to a ramp-position input.

Example A

The motor pole is located at $s = 23$. Threshold accuracy requirements demand that $K_1 K_2 = 10^5$. K_1 is set at 100, and K_2 is set at 1000. It is required that $\zeta = .7$. Find K_t .

$$K_t = \frac{2\zeta\omega_N - P}{K_2} = \frac{2a\omega_N - P}{K_2}$$

$$\omega_N^2 = 10^5 \quad \omega_N = 316$$

$$a = \zeta = .7$$

$$K_t = \frac{2(.7)(316) - 23}{1000}$$

$$K_t = \frac{420}{1000} = 0.42 \text{ VOLTS/RADIAN/SECOND}$$

Example B

The requirements are the same as in Example A; however, the maximum value of K_t which is obtainable is 0.32 volts per radian per second.

K_2 and K_1 are to be found.

$$K_2 = \frac{2 \text{ fwn} - P}{K_t} = \frac{2(.7)(316) - 23}{0.32}$$

$$K_2 = \frac{420}{0.32} = 1310$$

$$K_1 K_2 = 10^5, \text{ THEREFORE:}$$

$$K_1 = 76.5 \text{ VOLTS/VOLT}$$

Example C

The requirements are again as specified in Example A; however, the additional restriction is added that the steady-state-position error to a ramp-position input of 1 radian per second must be less than 0.001.

Find K_t . From the steady-state error specification, it is found that:

$$K_v \geq 1000$$

If the K_v requirement is to be met, ω_N must be greater than 316. The compensation then depends on how large ω_N may be without exceeding the maximum bandwidth requirement. In this case, assume that there are no bandwidth requirements to be met.

$$K_v = \frac{\omega_N}{25} \quad \omega_N = 25 K_v = 1400$$

$$K_1 K_2 = (1400)^2 = 1.96 \times 10^6$$

$$\text{LET } K_1 = 10^3, \quad K_2 = 1.96 \times 10^3$$

$$K_t = \frac{25\omega_N - P}{K_2} = \frac{2(.7)(1400) - 23}{1.96 \times 10^3}$$

$$K_t = \frac{1960 - 23}{1.96 \times 10^3} = 0.99$$

volts per radian per second

Example D

Suppose that the requirements are the same as in Example C, but K_t is restricted to a maximum value of 0.5 volts per radian per second.

Find the values for K_1 and K_2 .

$$K_2 = \frac{25\omega_N - P}{K_t} = \frac{2(.7)(1400) - 23}{.5}$$

$$K_2 = \frac{1937}{.5} = 3874 \text{ RADIANS/VOLT}$$

$$K_1 = 505 \text{ VOLTS/VOLT}$$

Example E

The threshold requirements for a system dictate that $\omega_N = 10$, that is, $K_1 K_2 = 100$. The steady-state error requirements are such that $K_v \geq 100$. It is also required that $\zeta = .7$. The motor pole is located at $s = -20$. From the nomogram it is seen that in order that the steady-state-error specifications may be met, ω_N must be at least 140 radians/second. Again if no requirements are specified concerning maximum bandwidth, the system may be compensated. Suppose that this is the case and $\omega_N = 140$ radians/second. K_1 is set at 196 volts/volt, and K_2 at 100 radians/volt. The next step is the evaluation of K_t from equation (3-5).

$$K_t = \frac{2\zeta\omega_N - P}{K_2} = \frac{2(.7)(140) - 20}{100}$$

$$K_t = \frac{196 - 20}{100}$$

$$= 0.176 \text{ VOLTS/RADIAN/SECOND}$$

Example F

The requirements are as in Example E except that the steady-state-error requirement has been relaxed so that $K_v \geq 10$. Again the nomogram is used and it is found that for satisfactory steady-state behavior, $\omega_N = 14$. Again assuming no bandwidth specifications, K_1 is set at 19.6 and K_2 is set at 10 to make $\omega_N = 14$. Equation (3-5) is then solved for K_t as follows:

$$K_t = \frac{25 \omega_N - P}{K_2}$$

$$K_t = \frac{2(.7)(14) - 20}{10}$$

$$K_t = \frac{19.6 - 20}{10}$$

$$K_t = -0.04 \text{ VOLT/RADIAN/SECOND}$$

The minus sign indicates that in order to meet requirements, positive feedback must be used. Example F differs from the other examples in that the uncompensated system in Example F was too heavily damped, while the other systems were under-damped. This fact then suggests first-derivative feedback to correct the behavior of systems whose behavior is too sluggish as well as systems whose behavior is too oscillatory.

Next, consider a more practical case where more requirements are to be met simultaneously.

Example G

Consider a system which has the following specifications and parameters:

$$P = 10$$

$$K_v \geq 150 \text{ 1/second}$$

$$\omega_N \geq 50 \text{ radians/ second in order to meet threshold requirements}$$

$$\omega_N \geq 60 \text{ radians/ second in order to meet the requirement for overcoming a load disturbance}$$

$\omega_N \leq 80$ radians/second in order to satisfy bandwidth specifications

$$.4 \leq \zeta \leq .5$$

$\zeta \omega_N \geq 25$ 1/second in order to meet maximum settling-time requirements.

First use the nomogram to see if the requirements can be met using this compensation device. The $\omega_N \geq 60$ radians/second must be used for the lower limit because it meets both the threshold and load-disturbance requirements.

The area in which the ω_N requirement is satisfied is shown horizontally cross hatched in Figure 8. The vertically cross-hatched region represents the area for acceptable values of K_v . The obliquely cross-hatched region represents the area for acceptable values of ζ . Notice that the ω_N and K_v requirements, or the ω_N and ζ requirements, or the K_v and ζ requirements may be met, but all three requirements cannot be satisfied simultaneously. This is shown on the nomogram by the fact that the three areas have no regions in common. At this point, the designer is confronted by two possibilities: he may relax one or more of the specifications, or he may look for another compensation device. Assume in this case that the K_v requirement may be relaxed so that $K_v \geq 80$.

It is observed in Figure 9 that there is a region where all specifications may be met simultaneously. This region is shaded. One point within this region specifies roots such that:

$$K_v = 80$$

$$\omega_N = 80$$

$$\zeta = .5$$

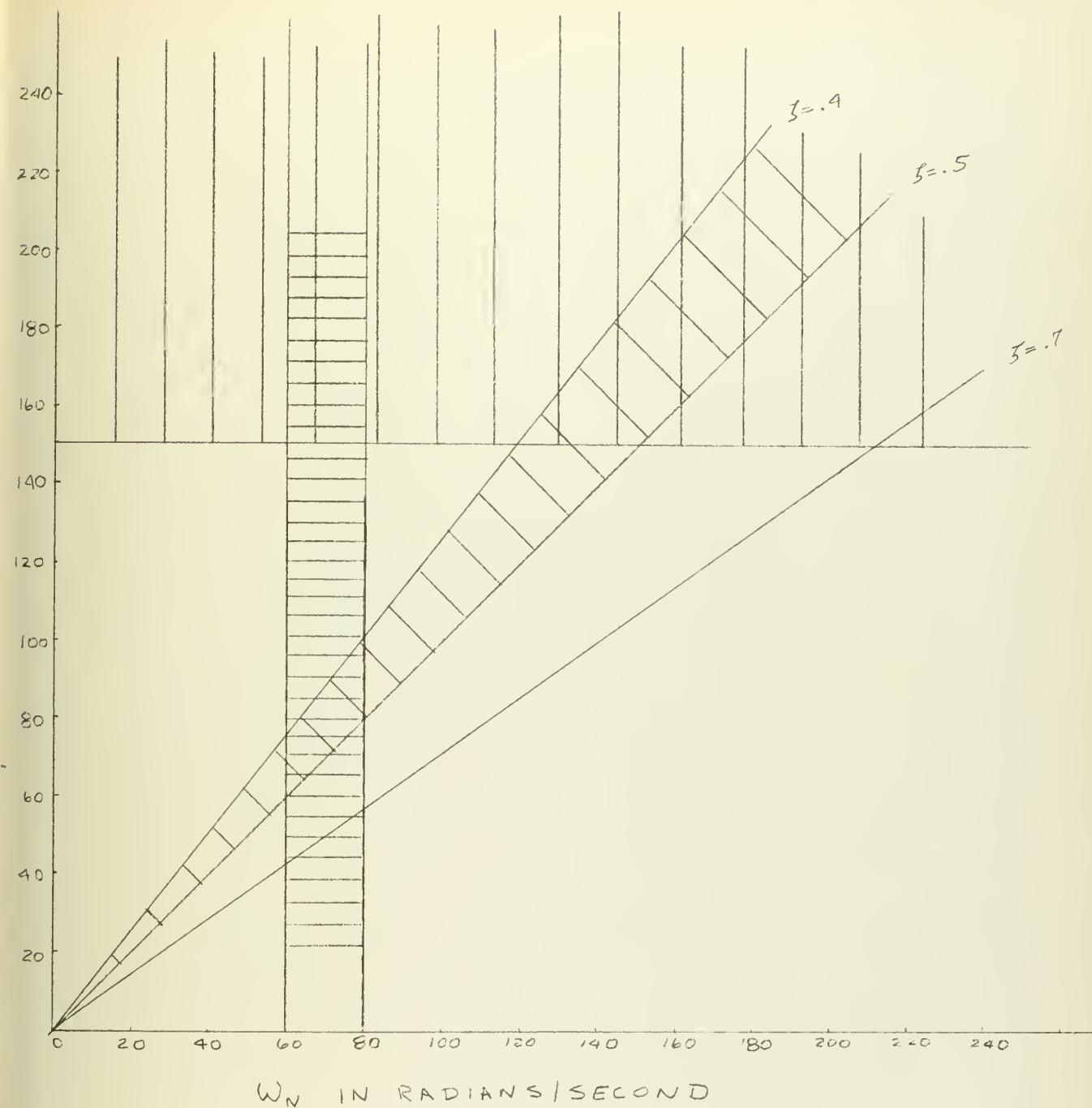


FIGURE 8 NOMOGRAM FOR EXAMPLE G

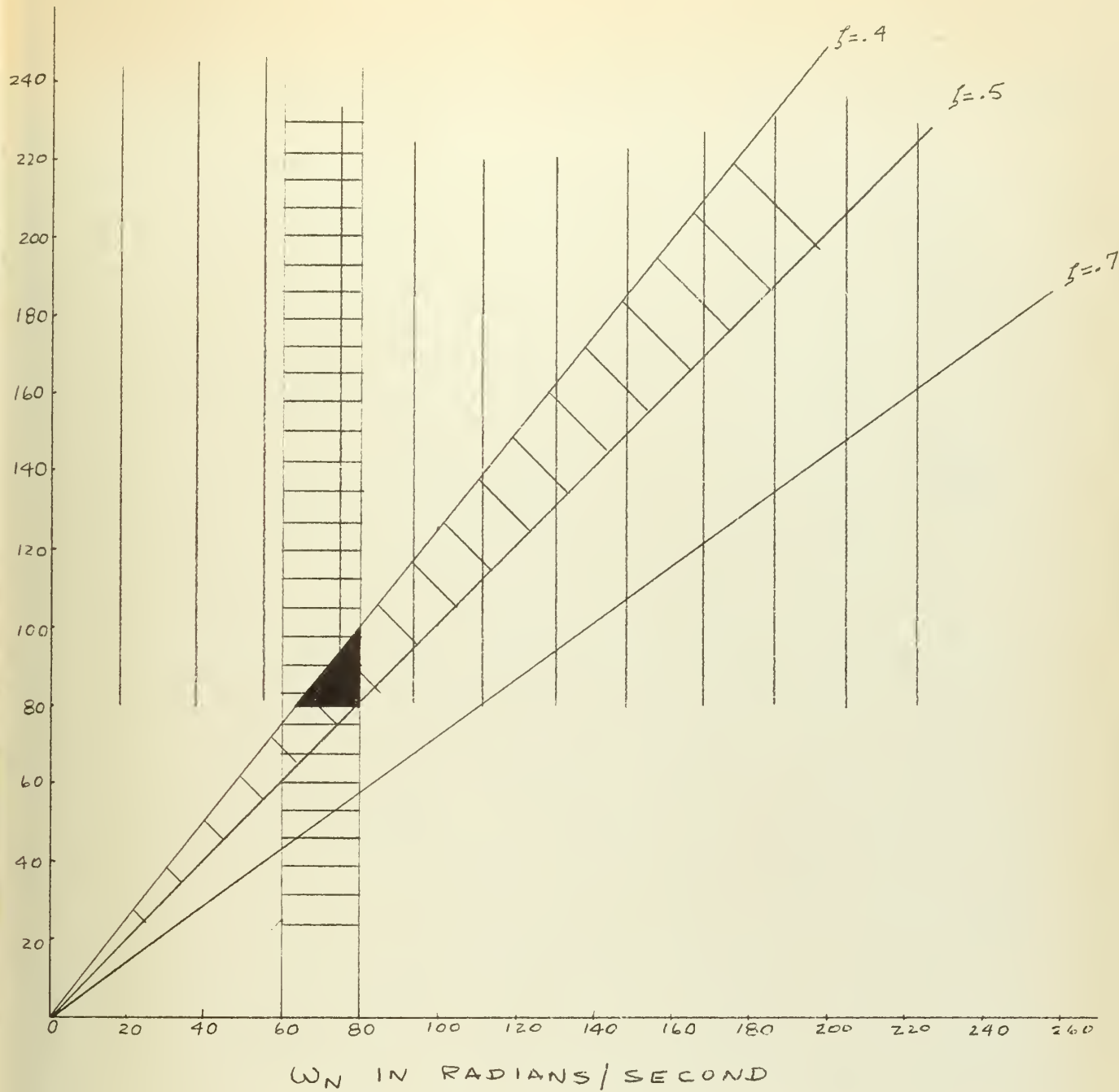


FIGURE 9 REVISED NOMOGRAM FOR EXAMPLE G

Note that the settling-time requirement that $\zeta \omega_N \geq 25$ is also satisfied.

$$\omega_N^2 = 6400$$

$$\text{LET : } K_1 = 64$$

$$K_2 = 100$$

$$K_t = \frac{2\zeta \omega_N - P}{K_2}$$

$$= \frac{2(.5)(80) - 10}{100}$$

$$K_t = \frac{70}{100} = 0.7 \text{ VOLT/RADIAN/SECOND}$$

The salient features of this procedure are as follows:

1. The nomogram shown in Figure 7 allows a rapid answer to the question of whether or not a system of the general form of Figure 1 can be compensated using first-derivative feedback.
2. Equation (3-5) may then be manipulated to yield the value for the tachometer gain or K_2 depending on the nature of the problem.
3. This technique may be extended to include systems where it may be desirable to feed the compensation signal around the entire system. This may be accomplished by setting K_1 equal to unity in the equations derived.

5. An Alternate Approach to the Second-order Problem

In the course of this investigation, another possible approach was also examined. This approach does not utilize the unique block-diagram manipulation; however, it is included to point out possibilities which exist for making use of a transformation of variable. Again assume a system of the form illustrated in Figure 1. The performance criteria are as previously described in Section 2. The root locus for the uncompensated system is as shown in Figure 4. Note that the uncompensated root locus has the same shape for every system which fits the generalized block diagram of Figure 1; however, each root locus will be unique for a given value of the motor pole, P . This fact suggests that one root locus can be constructed to describe the generalized system of Figure 1 and the parameter P used as a scaling factor. In order to accomplish this, a new complex variable is defined:

$$\begin{aligned}w &= f(s) = s/P \\&= a + jb\end{aligned}\tag{5-1}$$

By using this definition, the block diagram for the uncompensated system is obtained as shown in Figure 10. The root locus for the uncompensated system, after the transformation of variable has been performed, is shown in Figure 11.

It is noted that for all values of P , the root loci are identical. The location of the roots is determined by the gain, $K_1 K_2 / P^2$, which is a function of P . Next, assume tachometer-feedback compensation, i.e., $G_c(s) = K_t s$. By block-diagram reduction, Figure 12 is obtained.

The root loci, on the s plane, for the compensated and uncompensated systems are illustrated in Figure 13.

$$W = \frac{s}{a}$$

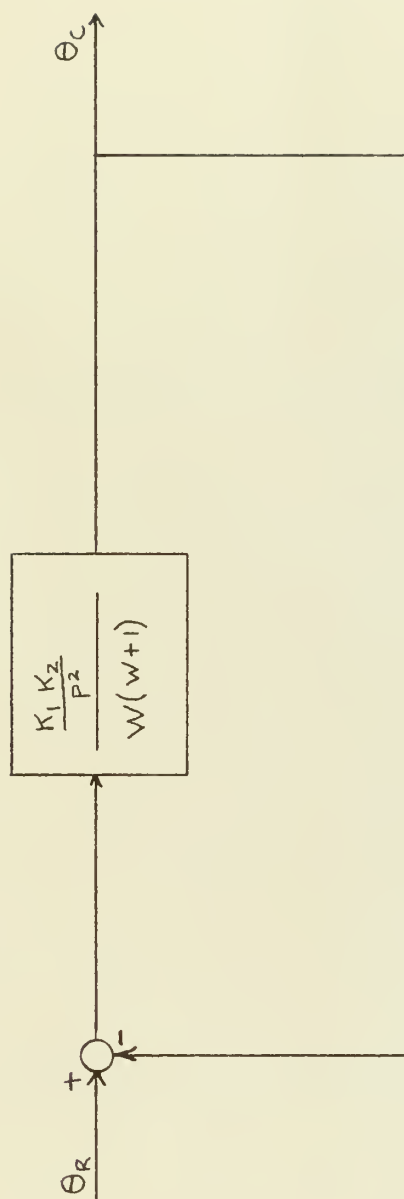


FIGURE 10 BLOCK DIAGRAM FOR THE UNCOMPENSATED SYSTEM OF FIGURE 1
AFTER A TRANSFORMATION OF VARIABLE

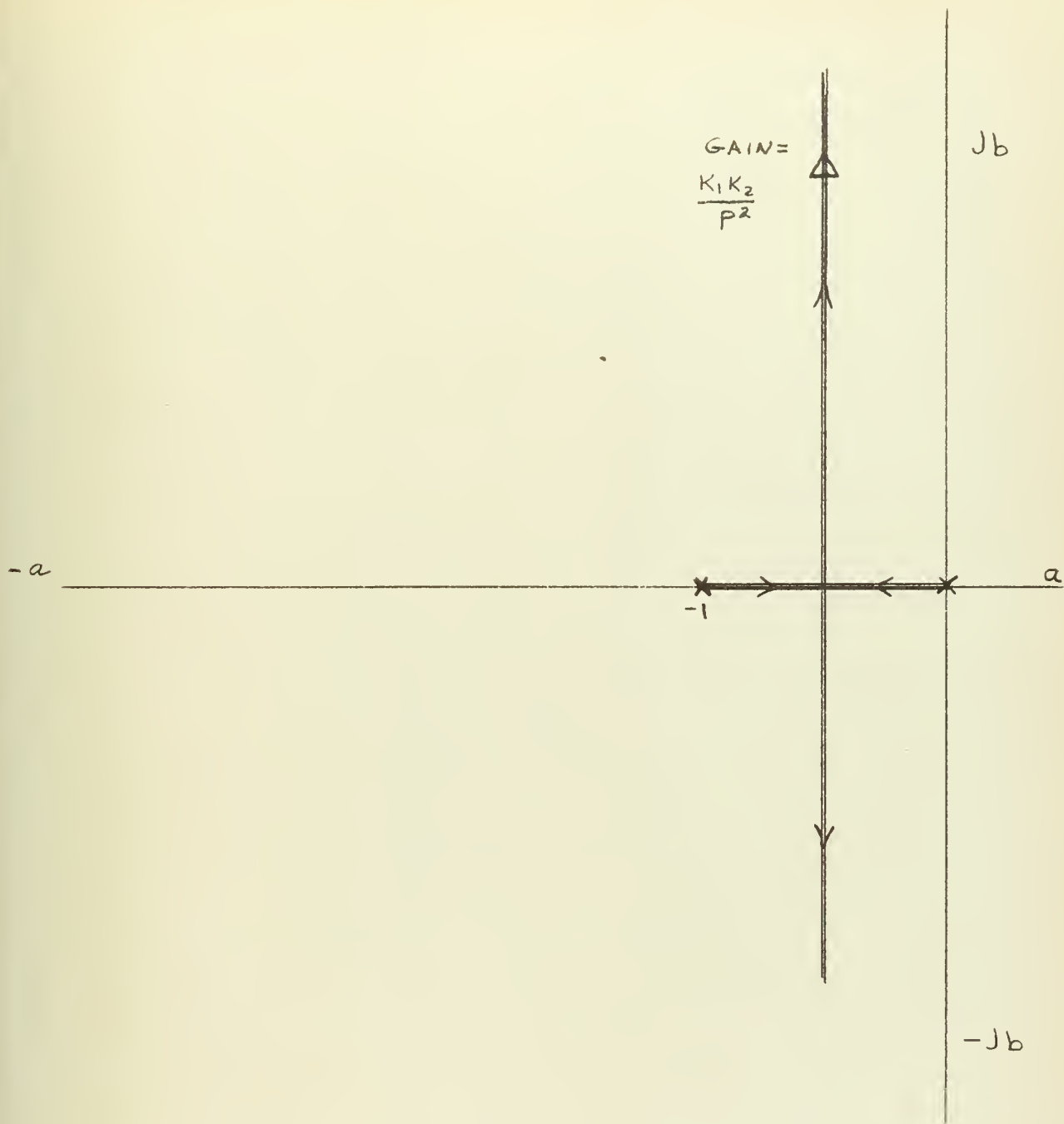


FIGURE 11 ROOT LOCUS OF UNCOMPENSATED SYSTEM ON w PLANE

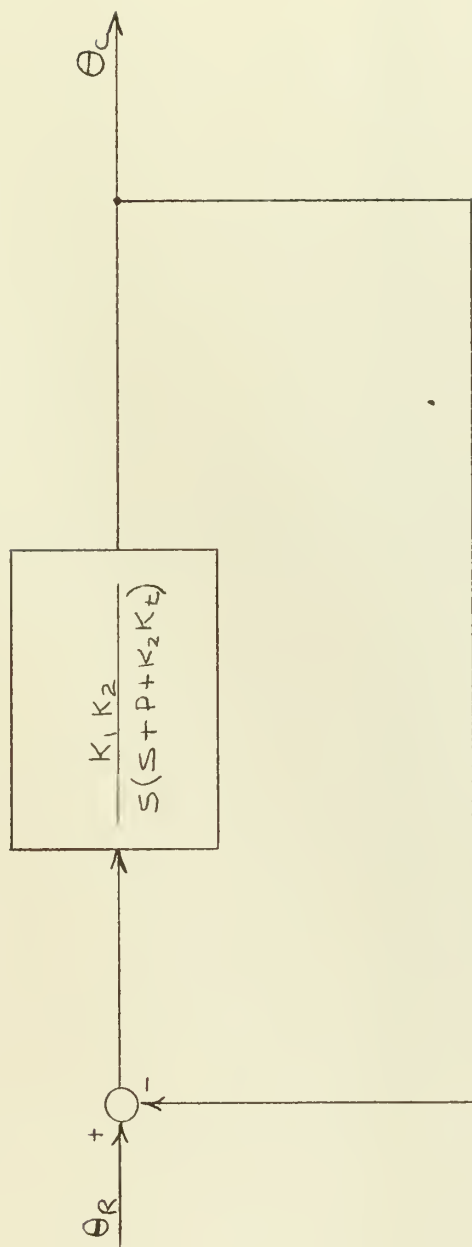


FIGURE 12 REDUCED BLOCK DIAGRAM OF THE COMPENSATED SYSTEM

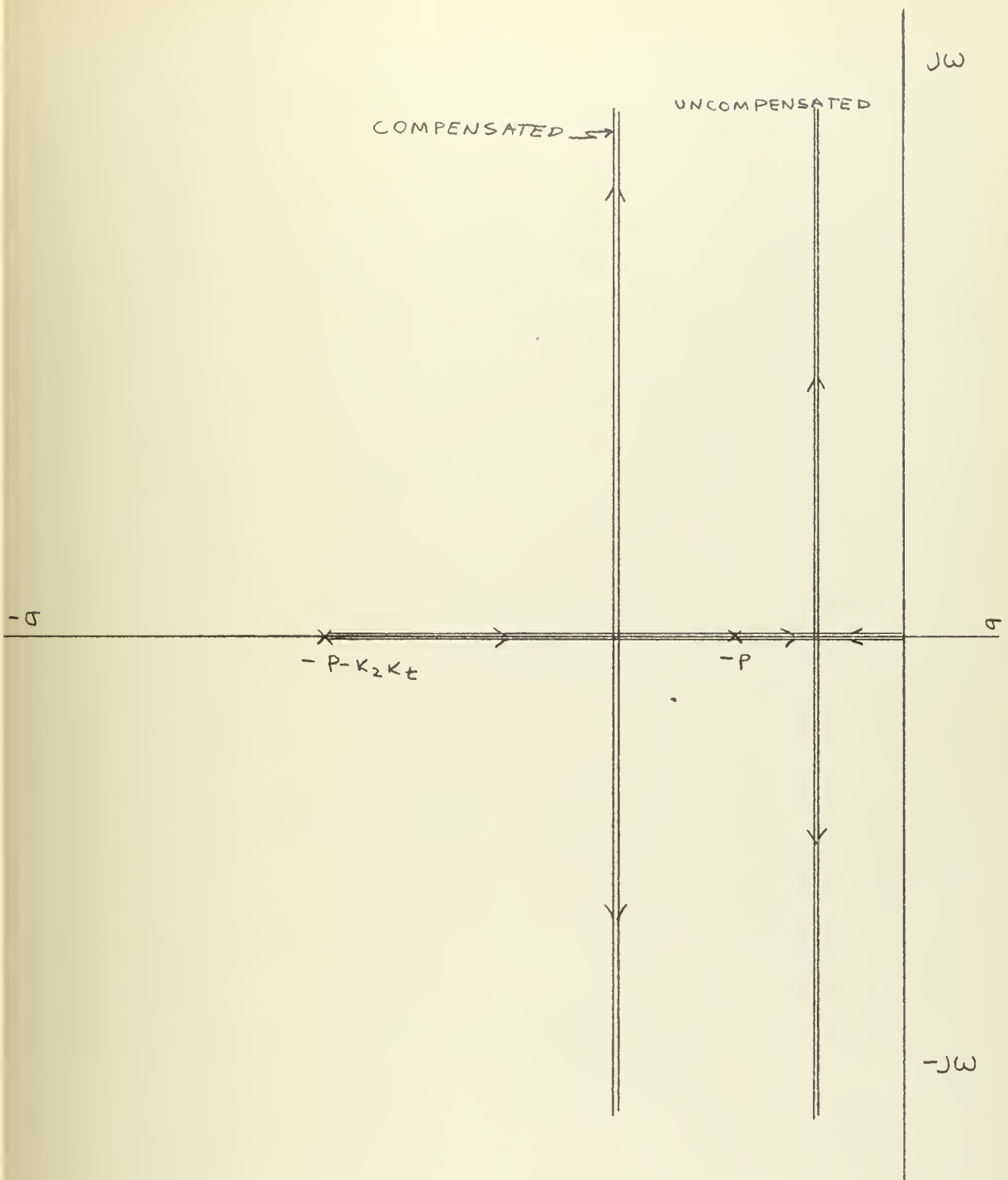


FIGURE 13 ROOT LOCI FOR THE COMPENSATED AND UNCOMPENSATED SYSTEMS ON THE S PLANE

If a transformation of variable is performed on the compensated system, the block diagram of Figure 14 is obtained. The root loci for the compensated and uncompensated systems are shown on the w plane in Figure 15.

The system specifications may be expressed in terms of the system parameters as in equations (3-8) through (3-11). These relationships are derived in the appendix. For the characteristic equation expressed in terms of the complex variable w , these quantities may be re-defined as (see appendix A5):

$$\omega_N(w) = \frac{1}{P} \sqrt{K_1 K_2} \quad (5-2)$$

$$\mathcal{J}(w) = \frac{P + K_2 K_t}{2 \sqrt{K_1 K_2}} \quad (5-3)$$

$$K_V(w) = \frac{1}{P} \frac{K_1 K_2}{P + K_2 K_t} \quad (5-4)$$

Therefore:

$$K_V(w) = \frac{\omega_N(w)}{2 \mathcal{J}(w)} \quad (5-6)$$

From the foregoing equations:

$$\frac{1}{P} \omega_N = \omega_N(w) \quad (5-7)$$

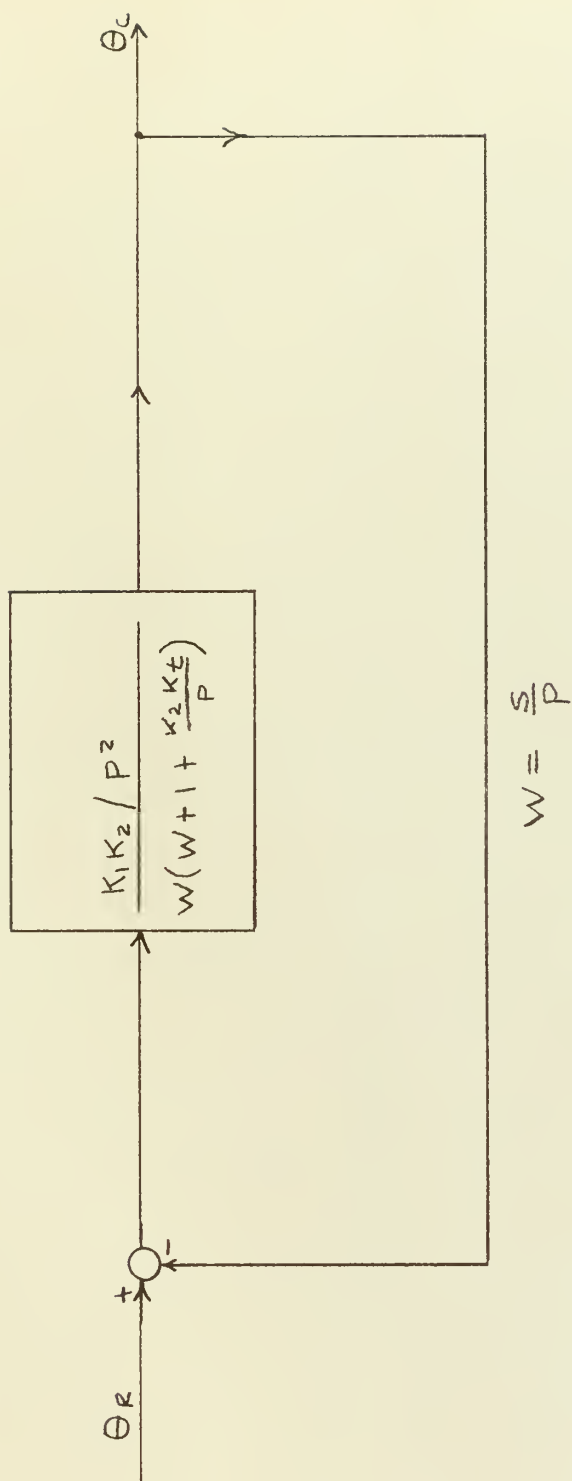


FIGURE 14 REDUCED BLOCK DIAGRAM OF THE COMPENSATED SYSTEM AFTER
A TRANSFORMATION OF VARIABLE

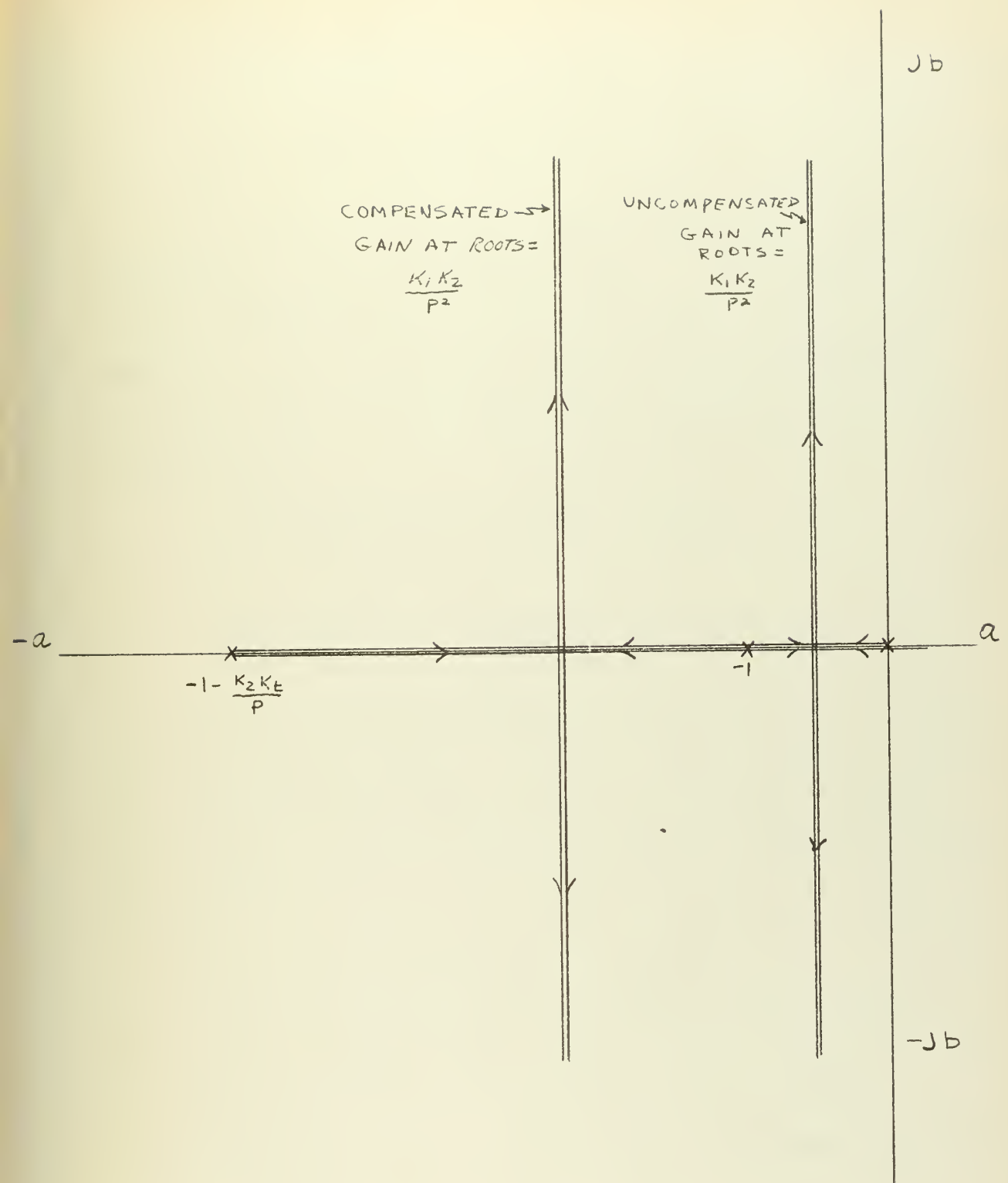


FIGURE 15 ROOT LOCI OF THE COMPENSATED AND UNCOMPENSATED SYSTEMS ON THE w PLANE

$$\zeta = \zeta(w) \quad (5-8)$$

$$\frac{1}{P} K_v = K_v(w) \quad (5-9)$$

It is observed that the system specifications expressed for the system as described in terms of the complex variable s may be readily transformed to expressions valid for w -plane coordinates. Next, construct a nomogram on the w plane. R_x is defined as being the desired root location on the w plane. It is observed from the root locus plot in Figure 15 that:

$$2 \operatorname{Re} \{R_x\} + 1 = \frac{K_2 K_t}{P} \quad (5-10)$$

where $\operatorname{Re} \{R_x\}$ denotes the real part of R_x .

It can also be derived (see appendix A6) that the loci of constant $K_v(w)$ are circles centered at $w = -K_v(w)$ with radii equal to $K_v(w)$. The relationship expressed in equation (5-10), the loci of constant $K_v(w)$, and the relationship expressed in equation (5-2) are illustrated in Figure 16.

The compensation technique is then as follows:

1. From the given specifications (converted to the w -coordinate system) determine a point or region on the w plane where it is desired to place a root of the characteristic equation.
2. Determine by inspection whether or not this method of compensation will yield the desired results.
3. Read $K_2 K_t$ from the nomogram and find K_t (or K_2 , or K_1).

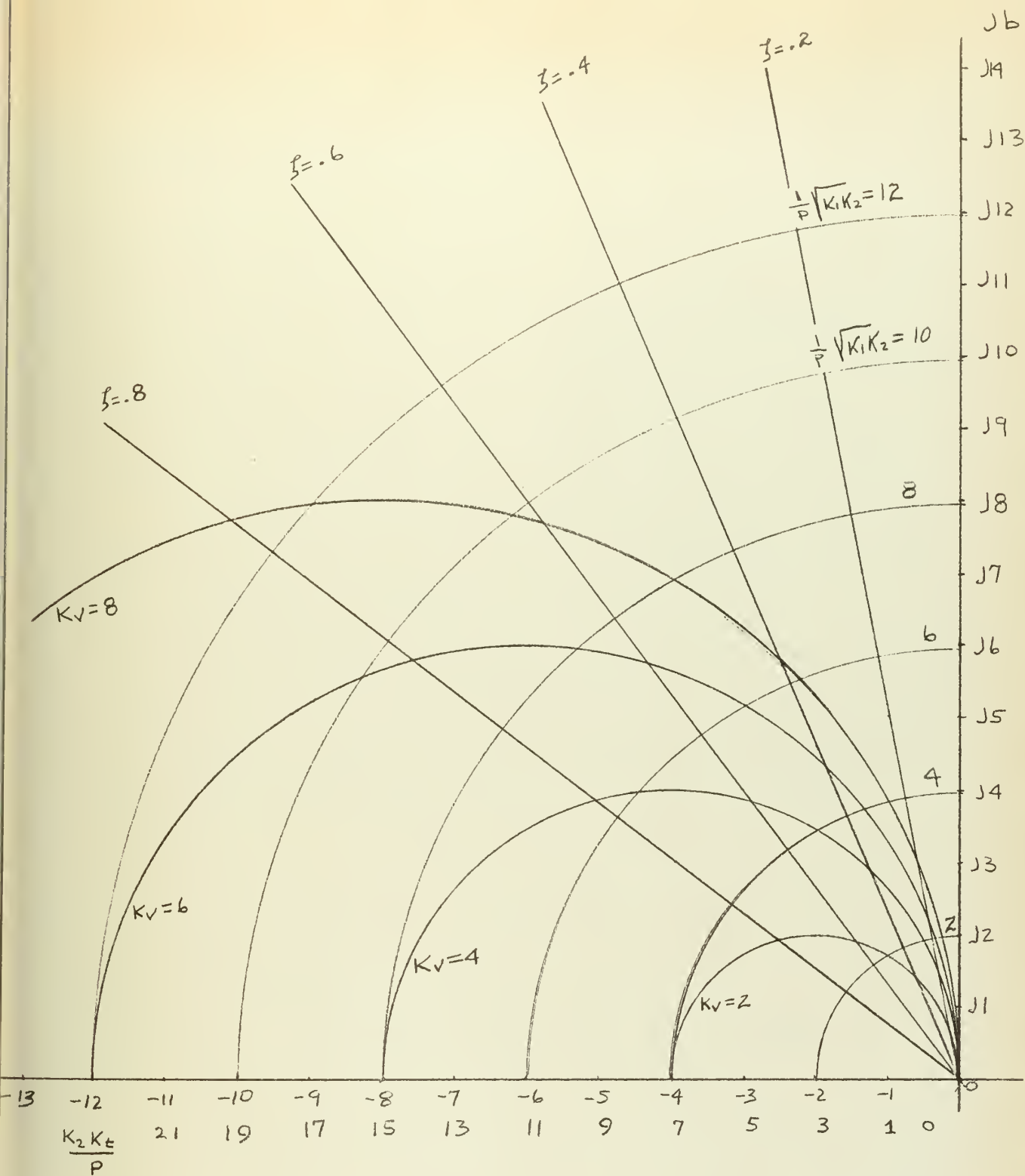


FIGURE 16 NOMOGRAM ON THE w PLANE

Consider Example G using this technique. The specifications after conversion to the w-plane system are:

$$\begin{array}{ll}
 K_V(w) \geq 15 & \\
 0.4 \leq \zeta(w) \leq 0.5 & \\
 \omega_N(w) \geq 5 & \text{from threshold requirement} \\
 \omega_N(w) \geq 6 & \text{from load-disturbance requirement} \\
 \omega_N(w) \leq 8 & \text{from bandwidth requirement} \\
 \zeta(w) \omega_N(w) \geq 2.5 & \text{from settling-time requirement}
 \end{array}$$

Next, define areas on the w plane using the above requirements. The range of $\omega_N(w)$, i.e., $6 \leq \omega_N(w) \leq 8$ defines an annular ring. The damping-ratio requirement restricts the desirable area to only a portion of this annular ring; however, note that the $K_V(w)$ requirement cannot be met if the $\zeta(w)$ and $\omega_N(w)$ requirements are satisfied. As in Example G, assume that the K_V requirement may be relaxed so that $K_V \geq 80$, i.e., $K_V(w) \geq 8$. Let $\zeta(w) = 0.5$, $\omega_N(w) = 8$, $K_V(w) = 8$

$$\begin{array}{ll}
 K_1 = 64 & \\
 K_2 = 100 & \text{since } \omega_N = 80
 \end{array}$$

From the nomogram shown in Figure 17, $K_2 K_t / P = 6.9$.

Therefore: $K_t = 69 / 100 = 0.69$ volts per radian per second.

This result verifies the result obtained in Example G.

The main disadvantage in this method is that tachometer-feedback compensation is assumed. It is also apparent that the simple relationships existent among K_V , ζ , AND ω_N are valid only for second-order systems. In fact, ζ AND ω_N are not defined for higher-order systems unless considerations of dominance are utilized to approximate these quantities.

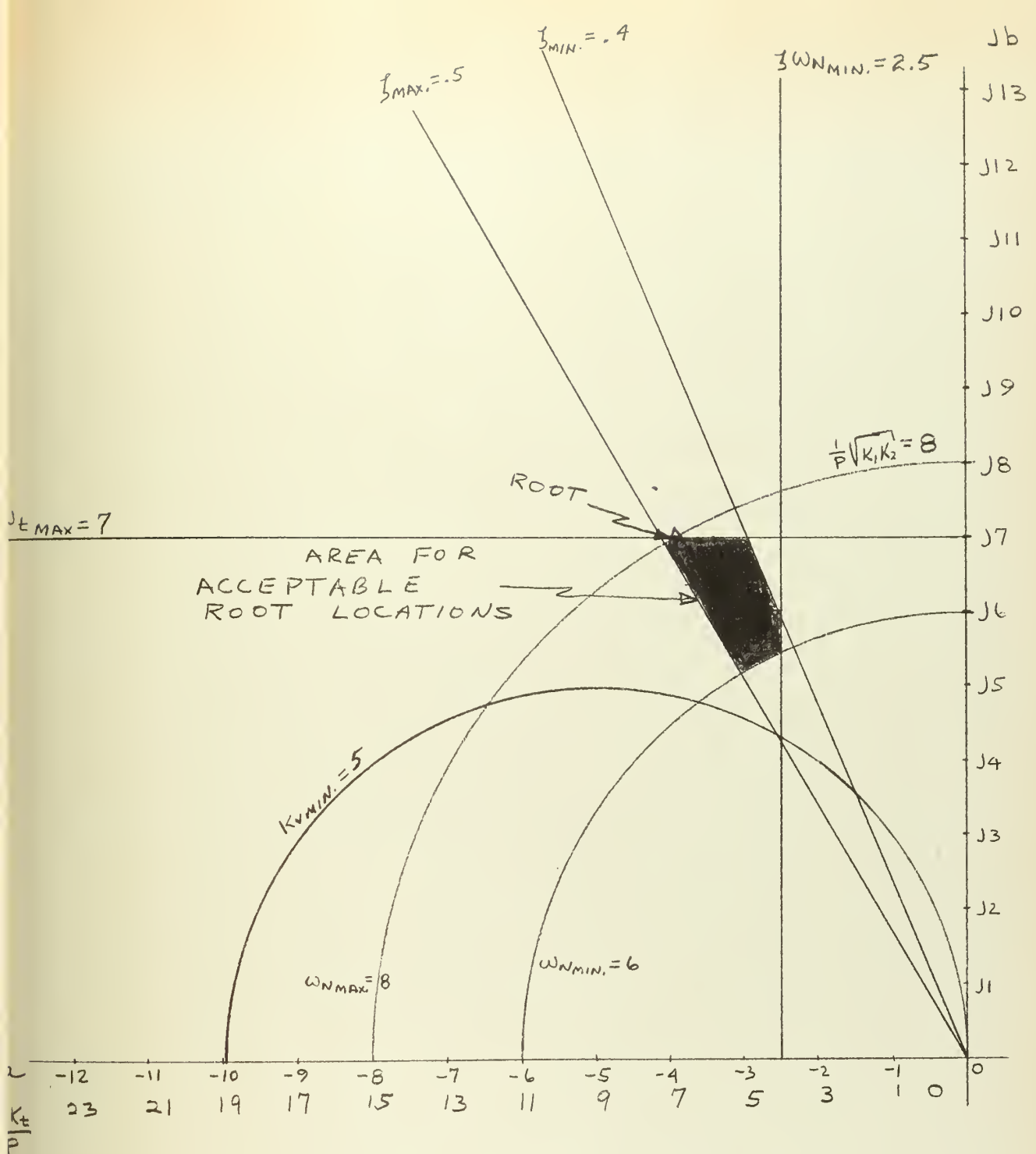


FIGURE 17 USE OF THE NOMOGRAM FOR COMPENSATION

Throughout the remainder of this investigation the block-diagram manipulation utilized earlier will be used to determine compensation possibilities.

6. Higher-order Systems

The same technique of block-diagram manipulation will be used to study the feasibility of tachometer-feedback compensation for systems which are of higher order than second order. The block-diagram manipulation will be performed to reduce the system to the form shown in Figure 18.

From Figure 18 it is seen that the loop transfer function is $G'(s)G_c(s)$. The root locus for the compensated system will then be found from the characteristic equation:

$$1 + G_c(s)G'(s) = 0 \quad (6-1)$$

The system to be considered is of the general form illustrated in Figure 1. The system will be investigated for each of the two possible positions of the compensation block.

Example A

$$\begin{aligned} \text{Let: } G_1(s) &= K_1 \\ G_2(s) &= K / s(s + a)(s + b) \end{aligned}$$

$G_2(s)$ may be either a two-phase motor, or a small d-c motor which may have either constant armature current or constant field current and with an armature inductance which is not negligible. From Figure 2 it may be found that:

$$\frac{\Theta_c}{B}(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)} \quad (6-2)$$
$$1 + G_c(s) \frac{G_2(s)}{1 + G_1(s)G_2(s)}$$

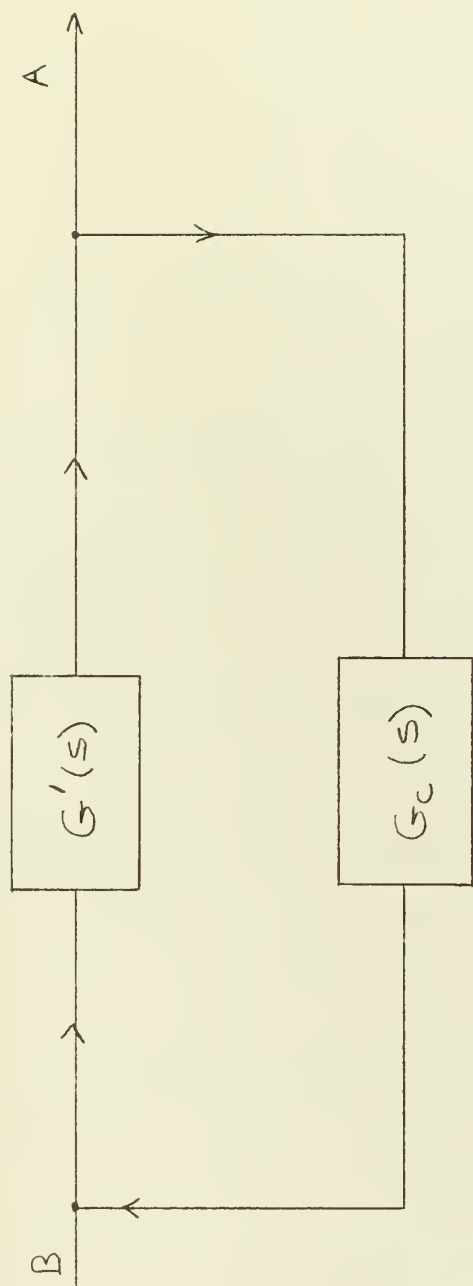


FIGURE 18 REDUCED BLOCK DIAGRAM IN GENERAL FORM

For $G_c(s) = 0$, the system is uncompensated and:

$$\frac{\Theta_c}{B}(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)} \quad (6-3)$$

The characteristic equation of the uncompensated system is:

$$1 + G_1(s)G_2(s) = 0 \quad (6-4)$$

or

$$\frac{K_1 K}{s(s+a)(s+b)} = -1 \quad (6-4a)$$

The root locus for the uncompensated system is shown in Figure 19.

Assume that the gain, $K_1 K$, is large enough to place two complex-conjugate roots in the right-half plane. It is desired to compensate the system using tachometer feedback.

If $G_c(s) = K_t s$, the characteristic equation of the compensated system is then:

$$1 + G_c(s) \frac{G_2(s)}{1 + G_1(s)G_2(s)} = 0 \quad (6-5)$$

or

$$G_c(s) \frac{K}{s(s+a)(s+b) + K_1 K} = -1 \quad (6-5a)$$

But the zeros of $s(s+a)(s+b) + K_1 K$ are the roots of the uncompensated system. The root locus for the compensated system is illustrated in Figure 20. Notice that if $|R_2| > |R_1 + R_1^*|$, the asymptote centroid will lie in the left-half plane and there will be a range of values of K_t

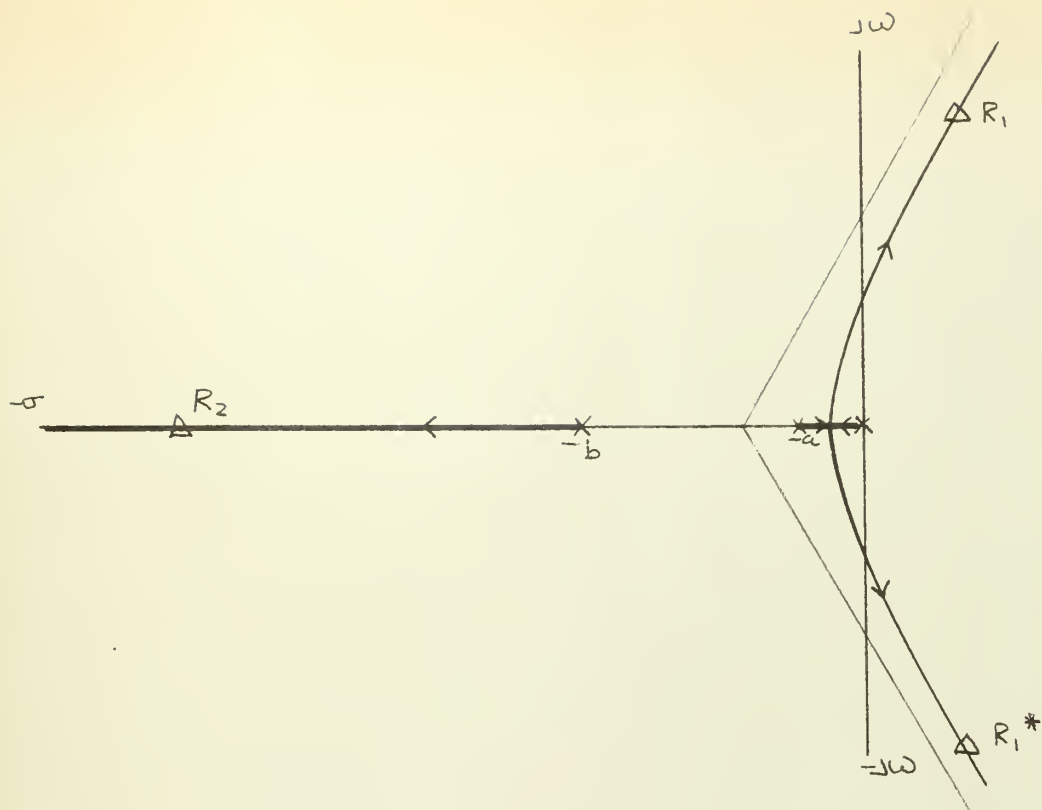


FIGURE 19 ROOT LOCUS FOR THE UNCOMPENSATED SYSTEM OF EXAMPLE A

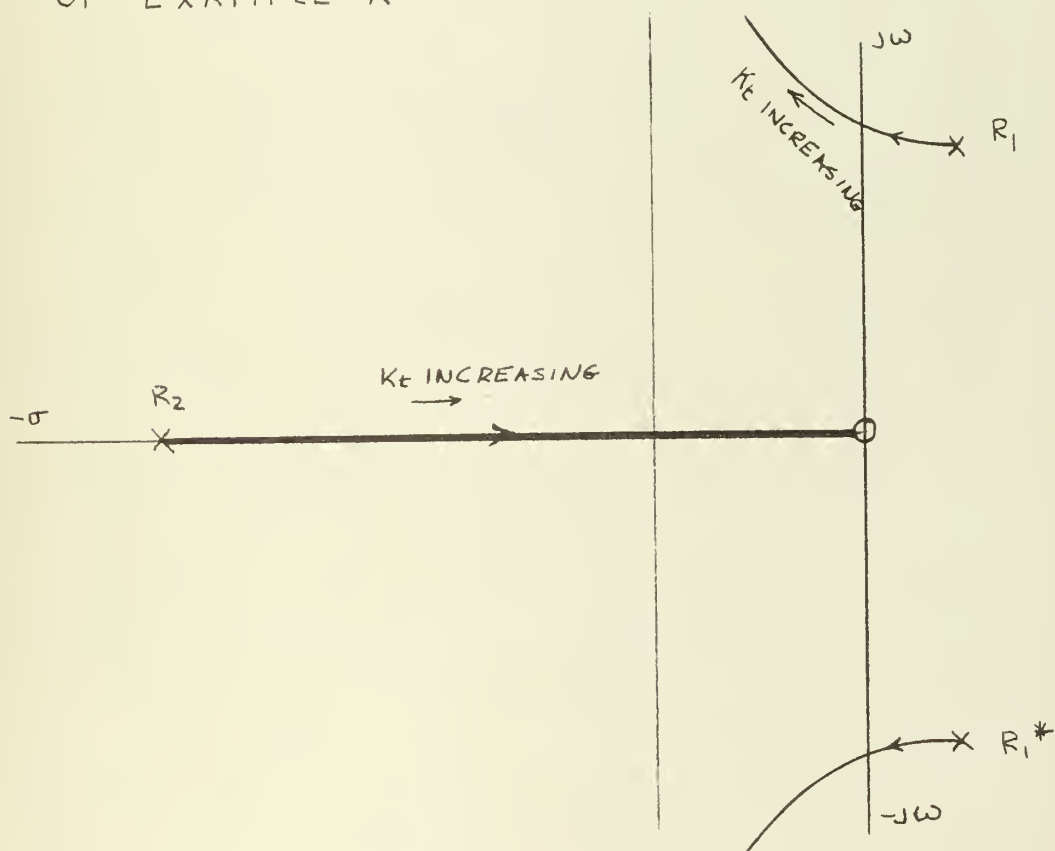


FIGURE 20 ROOT LOCUS FOR THE COMPENSATED SYSTEM OF EXAMPLE A

for which the system will be stable. The maximum value of damping obtainable may be determined by drawing a line from the origin tangent to the segments of root locus which terminate on zeros at infinity. In general, the oscillation frequency determined by the complex-conjugate roots will be rather large. It is quite possible for the value of K_t to reach a magnitude where the real root may dominate the transient response. For this reason, consideration of dominance must not be neglected.

As a result of this qualitative examination it would be concluded that tachometer-feedback compensation may be acceptable only if the asymptote centroid lies in the left-half plane. If this condition is met, system stability can be ensured for a range of values of K_t . The damping ratio, transient oscillating frequency, and dominance determine whether or not the desired transient response can be obtained.

By using tachometer feedback, the type number of the system is not altered. The velocity error constant, however, is affected by the value of K_t necessary for desired transient response. This fact is illustrated in the equations below. For the uncompensated system:

$$K_V = \lim_{s \rightarrow 0} s \frac{K_1 K}{s(s+a)(s+b)} = \frac{K_1 K}{ab} \quad (6-6)$$

For the compensated system:

$$K_V = \lim_{s \rightarrow 0} s \frac{K_1 K}{s(s+a)(s+b) + K_2 K_t s} \quad (6-7)$$

$$K_V = \frac{K_1 K}{ab + K_2 K_t}$$

The same conclusions also apply if the motor is of greater than fractional-horsepower size in which case $G_1(s)$ becomes, in general:

$$\frac{K}{s(s+a+jb)(s+a-jb)} = G_1(s) \quad (6-8)$$

Example B

Next, let: $G_1(s) = \frac{K_1}{s+a}$

$$G_2(s) = \frac{K_2}{s(s+b)}$$

This system has the blocks $G_1(s)$ AND $G_2(s)$ which may represent a Ward-Leonard System. This system is very similar to the system considered in Example A; however, the important difference lies in the fact that a compensation signal can be fed in between the blocks. As found from the block diagram in Figure 2, the characteristic equation of the uncompensated system is:

$$1 + G_1(s) G_2(s) = 1 + \frac{K_1 K_2}{s(s+a)(s+b)} \quad (6-9)$$

The root locus for the uncompensated system is shown in Figure 21. Note that this is the same as for the uncompensated system in Example A.

With the addition of tachometer feedback, the characteristic equation becomes:

$$1 + G_c(s) \frac{G_2(s)}{1 + G_1(s) G_2(s)} = 0 \quad (6-10)$$

Therefore:

$$1 + K_t S \frac{K_2 (s+a)}{s(s+a)(s+b) + K_1 K_2} = 0 \quad (6-10a)$$

But the zeros of $s(s+a)(s+b) + K_1 K_2$ are the roots of the uncompensated system; therefore:

$$1 + \frac{K_2 K_t S (s+a)}{(s-R_1)(s-R_1^*)(s-R_2)} \quad (6-11)$$

The root locus for the compensated system is shown in Figure 22.

Notice that, contrary to the result obtained in Figure 20, no conditions must be placed on whether stability is attainable. That is, there is always a range of values of K_t for which the system will be stable. It also appears that all values of the damping ratio are attainable if no limits are placed on the magnitude of K_t . It is also apparent that a desirable damping ratio can be obtained for a smaller magnitude of transient oscillating frequency and a smaller magnitude of resonant frequency than was the case for the system in Example A. This means that, in general, this system can be compensated so that the damping ratio is satisfactory, and at the same time have a better opportunity of meeting transient-oscillating-frequency and bandwidth specifications. In addition, considerations of dominance are less difficult because the real root of the system moves farther away from the complex-conjugate roots as the damping ratio increases. Therefore, it would be anticipated that in general the complex-conjugate roots would dominate the transient response.

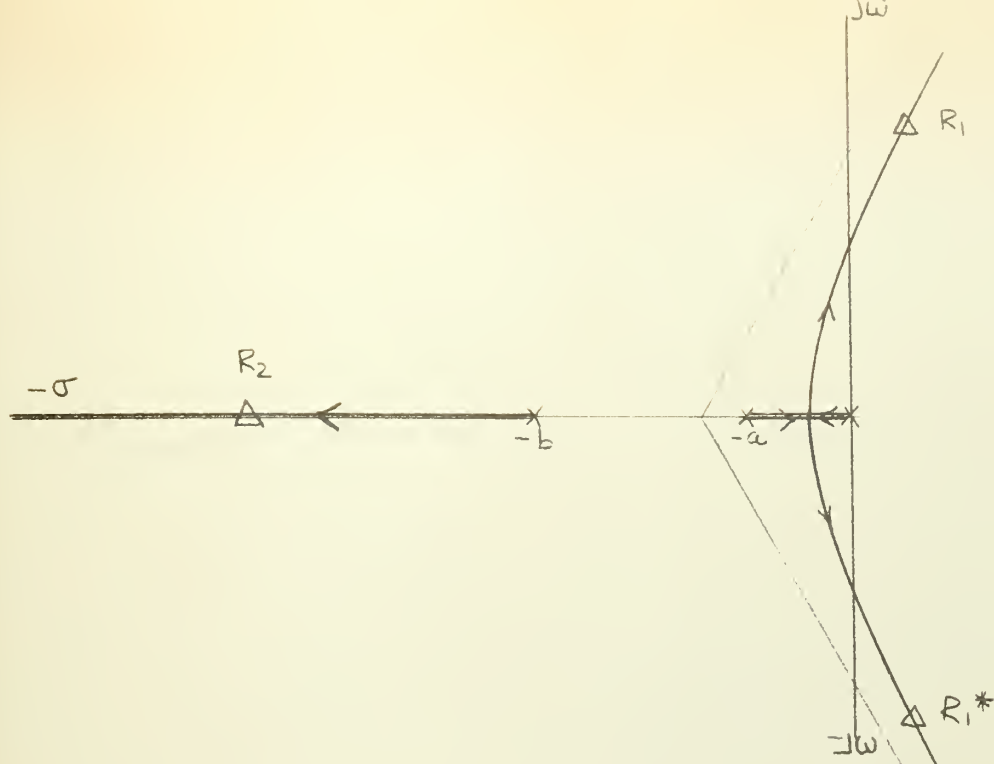


FIGURE 21 ROOT LOCUS FOR THE UNCOMPENSATED SYSTEM OF EXAMPLE B

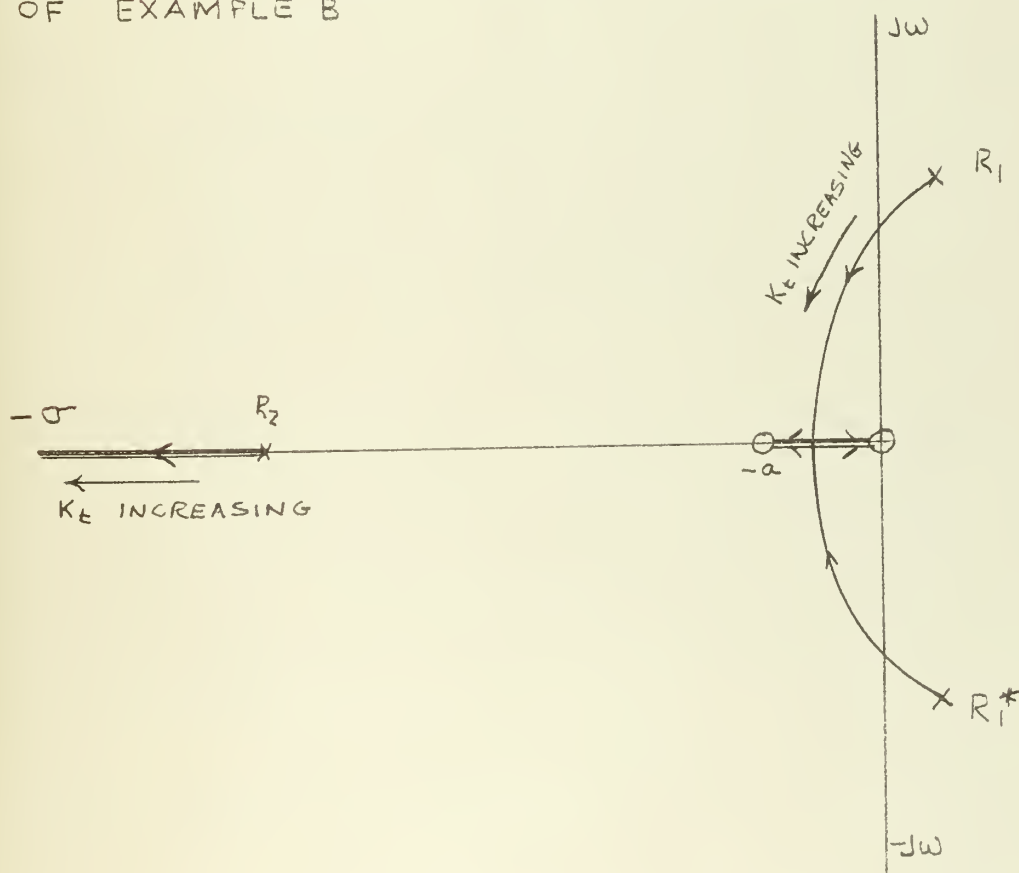


FIGURE 22 ROOT LOCUS FOR THE COMPENSATED SYSTEM OF EXAMPLE B

Once again it is found that the addition of the tachometer does not alter the type number of the system. The velocity-error coefficients are given below for both the compensated and uncompensated systems.

$$K_V = \frac{K_1 K_2}{a b} \quad (6-12)$$

For the compensated system:

$$K_V = \frac{K_1 K_2}{a(b + K_2 K_t)} \quad (6-13)$$

Therefore, in general, the steady-state accuracy for a ramp-position input signal will be poorer for the compensated system.

The root locus for this same system with $G_c(s)$ in path II is essentially the same as shown in Figure 20.

Example C

Let: $G_1(s) = \frac{K_1}{(s+a)}$

$$G_2(s) = \frac{K_2}{s(s+b)(s+c)}$$

Using the same techniques as employed before, the uncompensated root locus is found as shown in Figure 23. If a tachometer is added in path I, the root locus for the compensated system will be of one of the forms illustrated in Figure 24. The configurations shown in a) and b) of Figure 24 can occur only when:

$$|R_2 + R_2^*| > |R_1 + R_1^* + a| \quad (6-14)$$

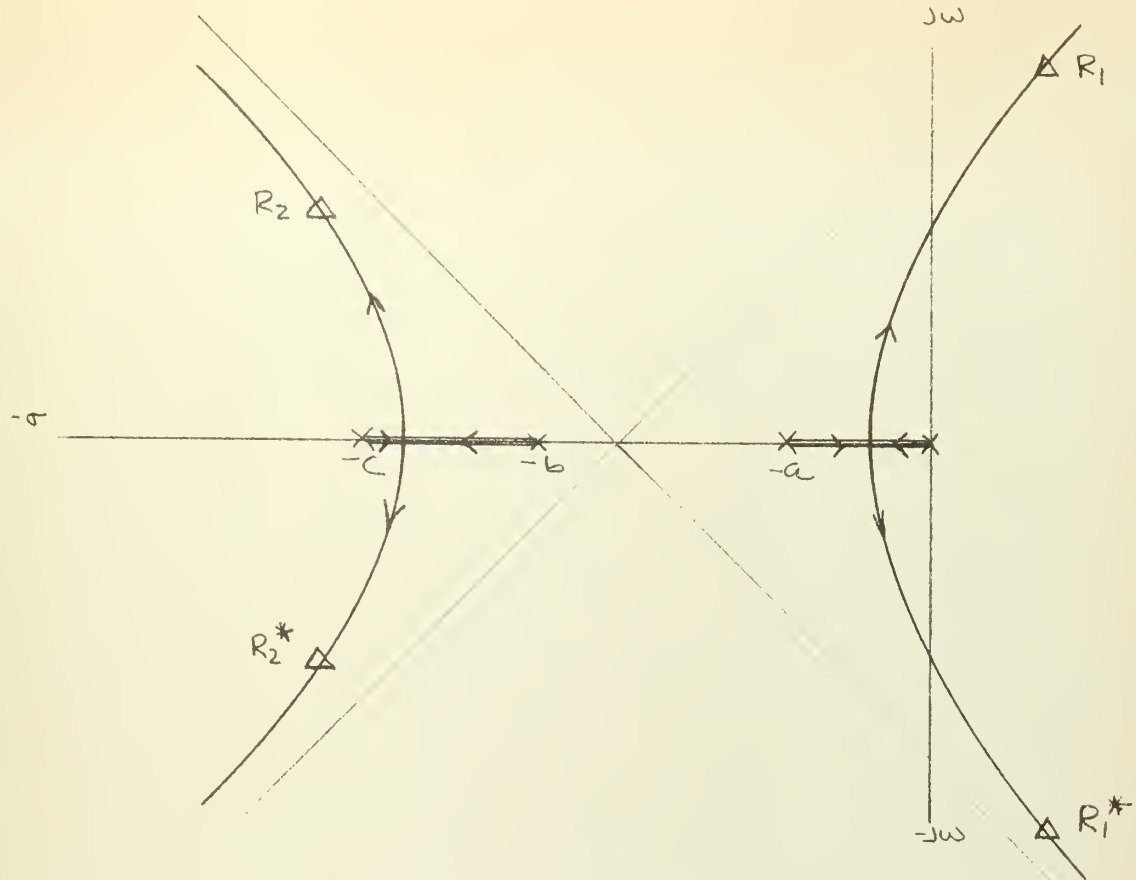


FIGURE 23 ROOT LOCUS FOR THE UNCOMPENSATED SYSTEM OF EXAMPLE C

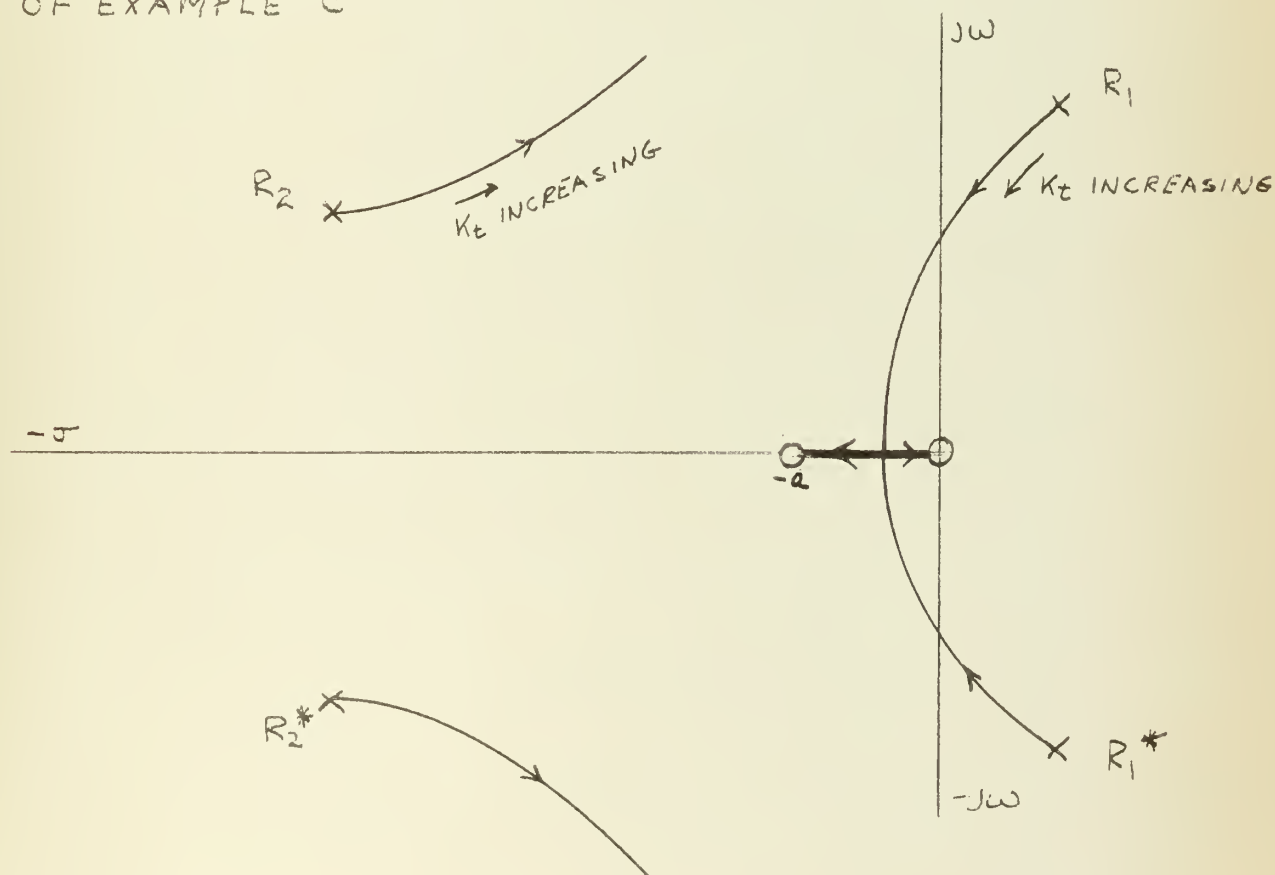


FIGURE 24 (a)



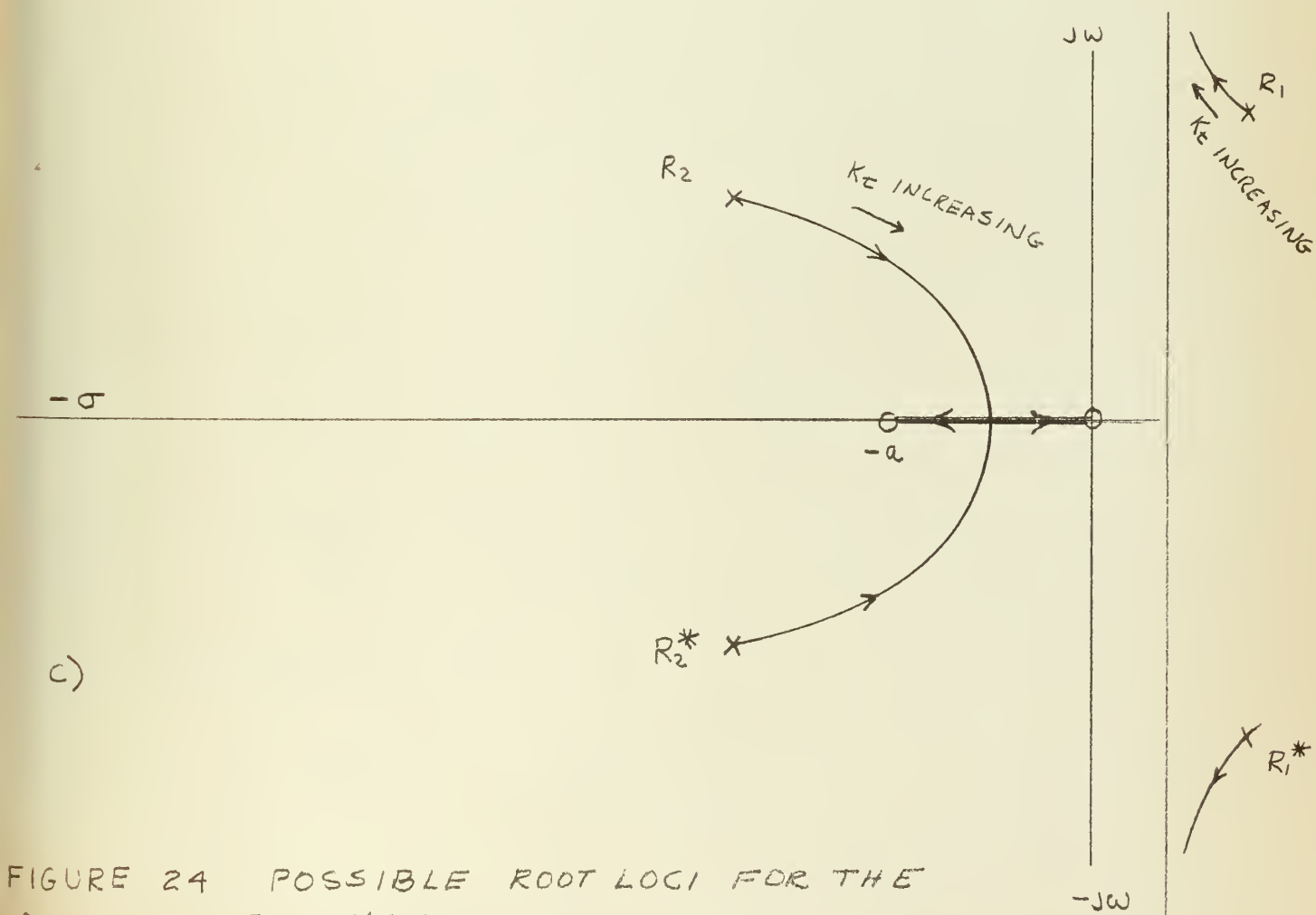
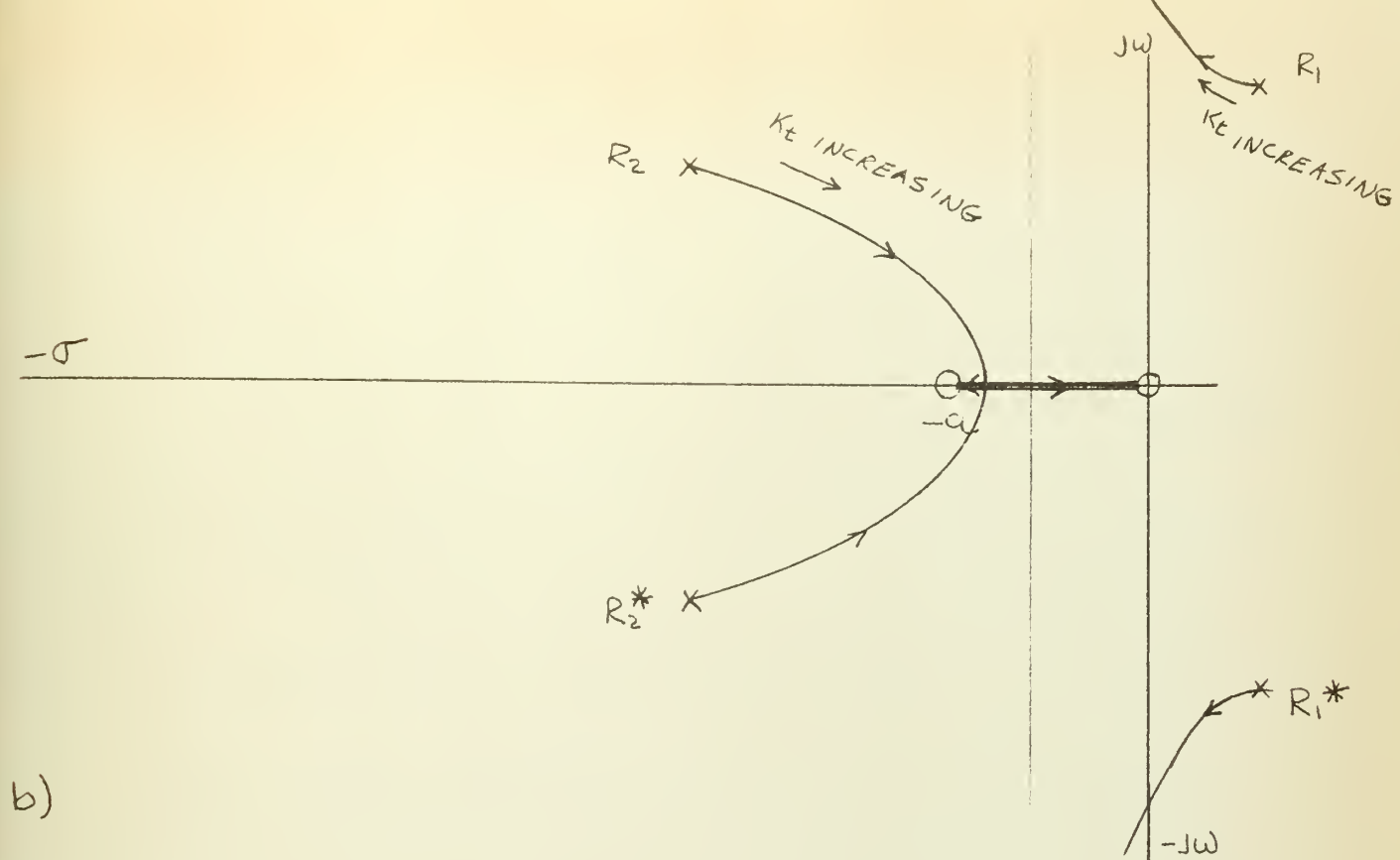


FIGURE 24 POSSIBLE ROOT LOCI FOR THE COMPENSATED SYSTEM OF EXAMPLE C

For each of these configurations there exists a range of values of K_t for which stability can be achieved. The configurations shown in a) and b) also indicate that it may be possible to achieve the desired response by using tachometer feedback; however, consideration must again be given to the question of dominance. It may happen that a value of K_t large enough to make one pair of complex-conjugate roots dominate the transient response may also make the damping ratio larger than desired and cause the system to be sluggish. The root locus shown in c) will occur when:

$$|R_2 + R_2^*| < |R_1 + R_1^* + a| \quad (6-15)$$

This system will be unstable for all values of K_t and therefore undesirable. It should be noted that the determination of the location of the asymptote centroid will depend entirely on the parameters of the uncompensated system. For this particular system, the complexity of the root loci makes it difficult to make a general statement concerning the feasibility of obtaining the desired performance by using tachometer feedback. It is shown, however, for some systems of this form, the use of tachometer feedback may be ruled out very rapidly. For other systems, the desired compensation may be achieved.

The system retains its type-one characteristic. The velocity-error coefficients are given below.

For the uncompensated system:

$$K_v = \frac{K_1 K_2}{a b c} \quad (6-16)$$

For the compensated system:

$$K_v = \frac{K_1 K_2}{a (b c + K_2 K_t)} \quad (6-17)$$

Therefore, the steady-state accuracy to a ramp-position input will suffer as a result of using tachometer feedback as illustrated in this example. The results obtained in Figure 24 are also representative of the case where $G_2(s)$ is of the form:

$$G_2(s) = \frac{K_2}{s(s+b+jc)(s+b-jc)} \quad (6-18)$$

When path II is used, the situation changes somewhat. The uncompensated root locus, as shown in Figure 23, remains the same. Three possibilities for the root locus of the compensated system are shown in Figure 25.

Obviously, the conditions shown in a) and b) of Figure 25 are completely unsatisfactory because the system will be unstable for all values of K_t .

The root locus shown in b) occurs when $|R_2 + R_2^*| < |R_1 + R_1^*|$

The root locus shown in c) shows that, under certain conditions, compensation can be achieved; however, it is also evident that for large values of K_t this system may also be unstable. The velocity-error coefficients are as given below.

For the uncompensated system:

$$K_V = \frac{K_1 K_2}{abc} \quad (6-19)$$

For the compensated system:

$$K_V = \frac{K_1 K_2}{abc + K_1 K_2 K_t} \quad (6-20)$$

Again it is observed that the introduction of tachometer feedback decreases steady-state accuracy to a ramp-position input signal.

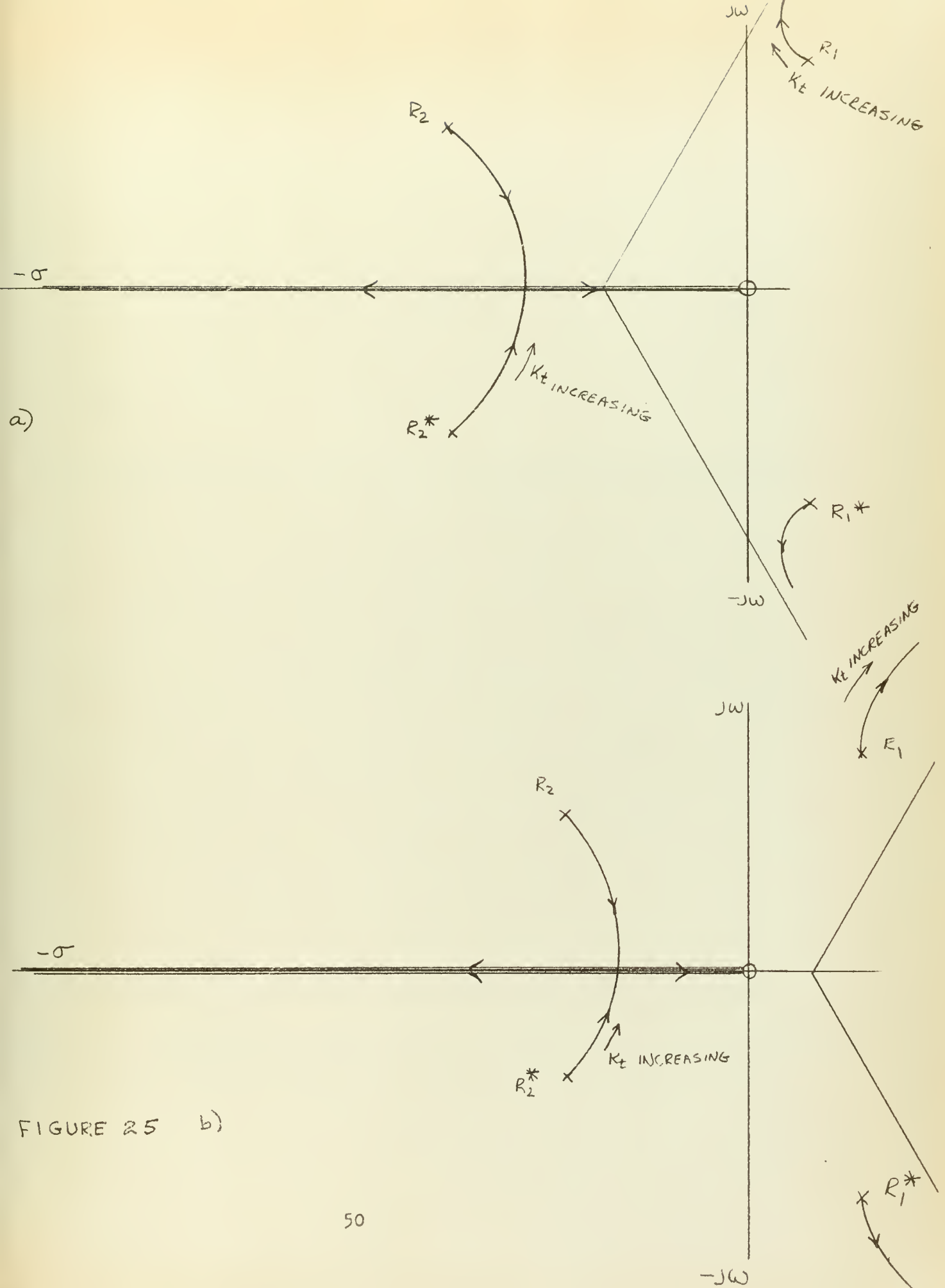
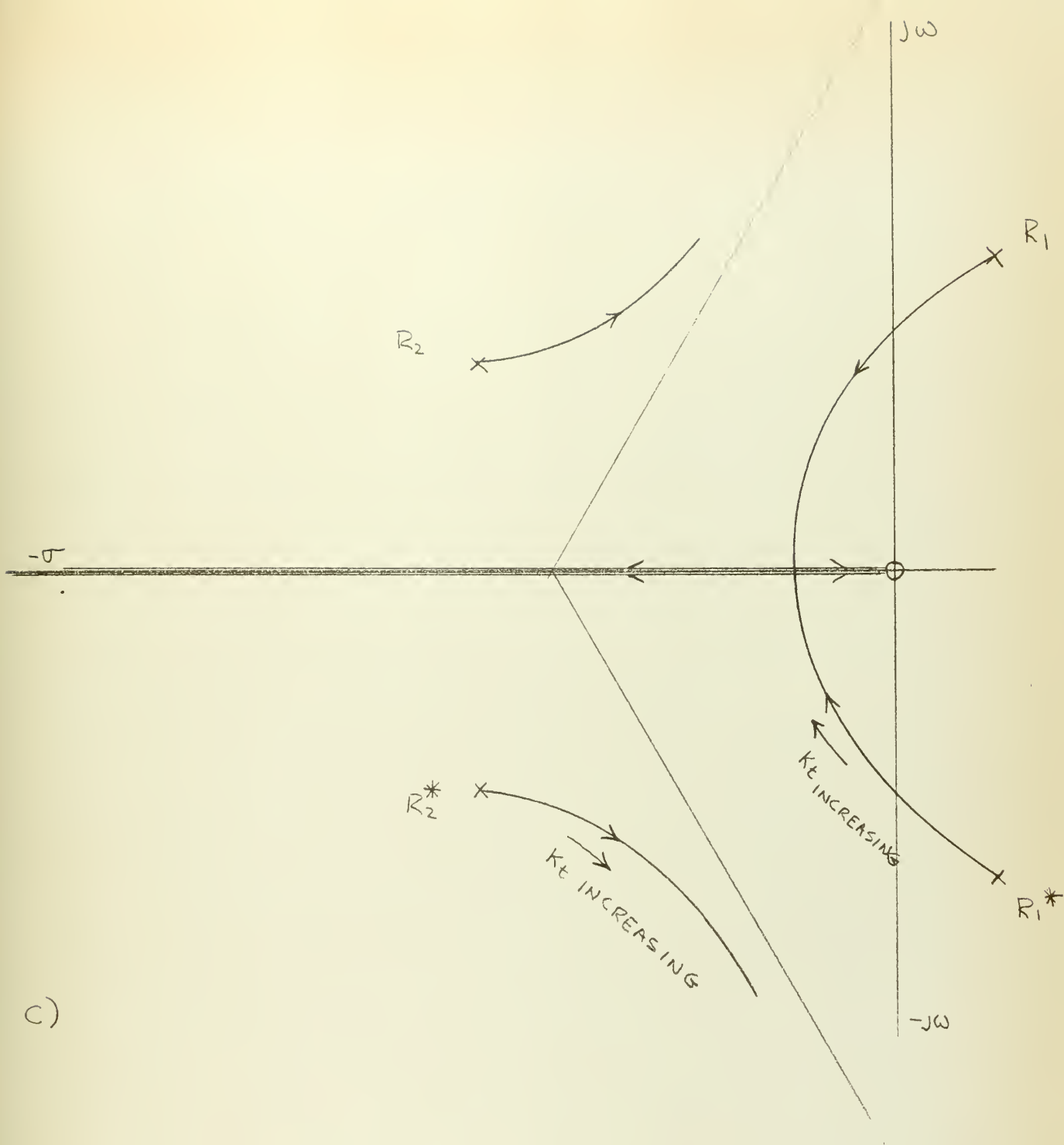


FIGURE 25 b)



c)

FIGURE 25 POSSIBLE ROOT LOCI FOR THE COMPENSATED SYSTEM OF EXAMPLE C (COMPENSATOR IN PATH II)

Example D

In the block diagram shown in Figure 1, let:

$$G_1(s) = \frac{K_1}{(s+a)(s+b)}$$

and

$$G_2(s) = \frac{K_2}{s(s+c)}$$

The root locus for the uncompensated system is of the form illustrated in Figure 23. This is the case because the uncompensated systems in both Example C and in this example have the same forward transfer function; however, in this example path I allows a signal to be fed into the system which is not fed around the block containing a pole at $s = -b$. In essence this means the effect is the same as if a compensation signal were introduced within the $G_2(s)$ block in Example C.

The resulting root locus for the compensated system is as shown in Figure 26. Notice that there will always be a range of values of K_t for which the system is stable. In addition, it appears likely that a value of K_t can be obtained which will set root positions so that satisfactory damping can be attained with reasonable values of transient oscillating frequency and resonant frequency. Determination of dominance appears to be relatively simple inasmuch as the zero farthest removed from the origin limits the closeness of the secondary pair of complex-conjugate roots. If the secondary roots are real, the location of Z_3 determines absolutely the magnitude of the secondary root closest to the primary roots. Comparing this result with Figure 24 it seems evident that the attempted compensation is aided by the ability to exclude the pole at $s = -b$ from the minor loop containing the compensation device.

The steady-state error coefficients are shown below.

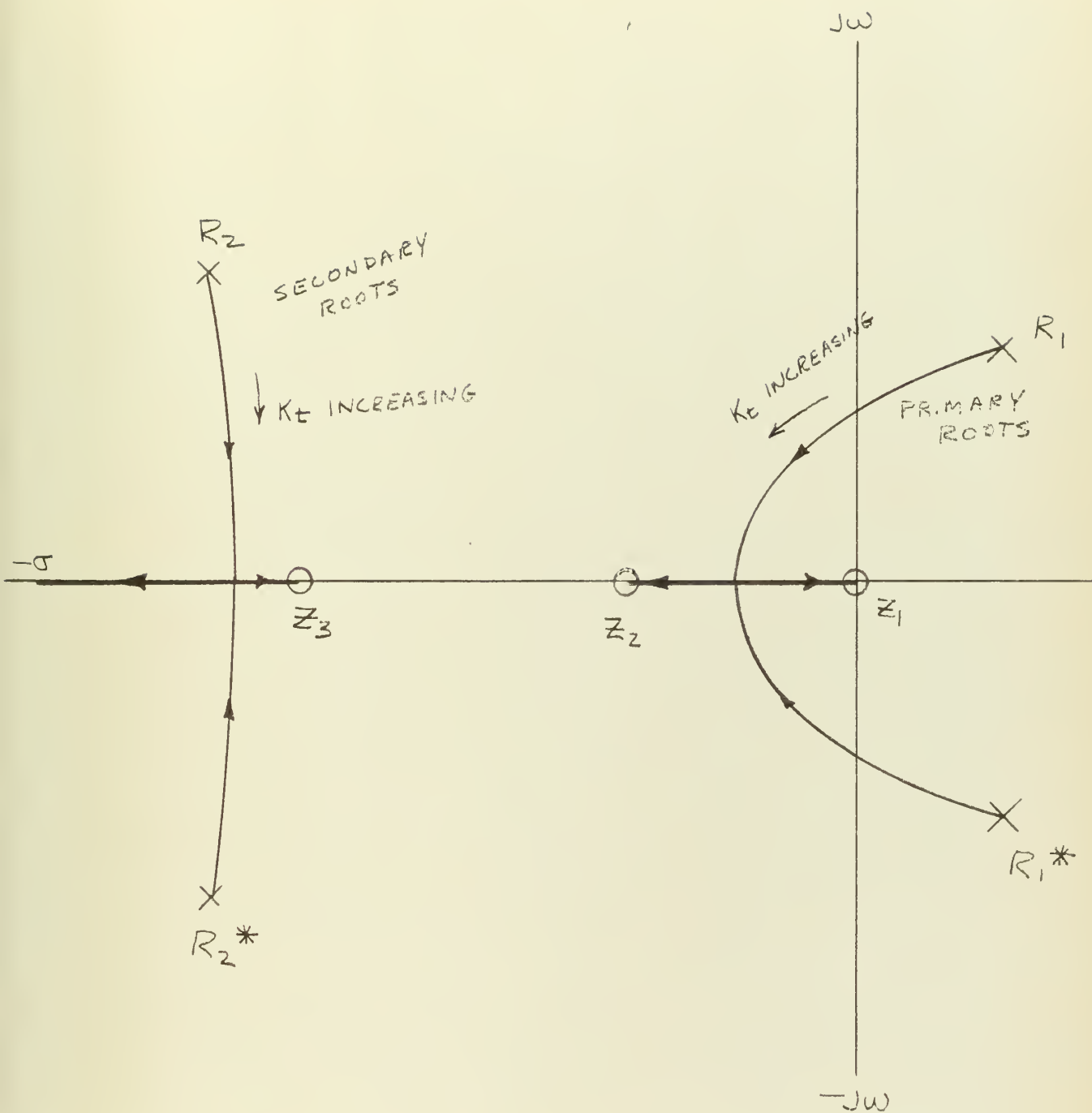


FIGURE 26 ROOT LOCUS FOR THE COMPENSATED SYSTEM OF EXAMPLE D

For the uncompensated system:

$$K_v = \frac{K_1 K_2}{a b c} \quad (6-21)$$

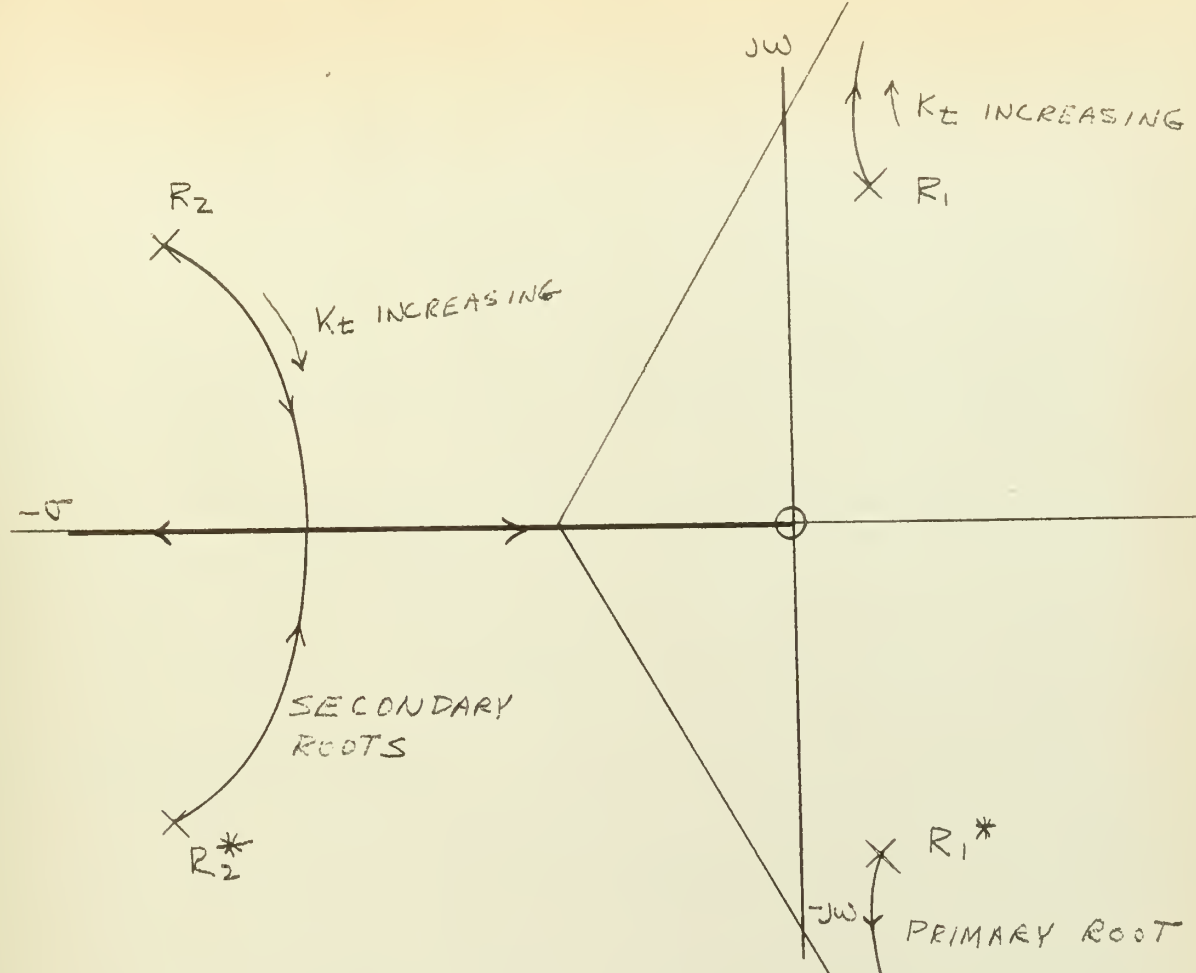
For the compensated system:

$$K_v = \frac{K_1 K_2}{a b (c + K_2 K_t)} \quad (6-22)$$

It is again noted that the use of tachometer feedback decreases the steady-state accuracy to a ramp-position input signal. Comparing equation (6-22) with equation (6-17) it is also observed that the increased likelihood of achieving satisfactory compensation results in less desirable steady-state characteristics.

Consider the same system with the tachometer contained in path II. The uncompensated root locus is the same as that shown in Figure 23. Two possibilities for the root locus of the compensated system are shown in Figure 27. The root locus shown in a) of Figure 27 obviously characterizes a system which is unacceptable; the system is unstable for all values of K_t . The root locus in b) of Figure 27 represents a system which may conceivably meet a rather loose set of specifications; however, ideal conditions must be obtained in order for a root locus plot as shown in b) to be the result of compensation. That is, R_1 must have a relatively small real part so that the asymptote centroid lies well into the left half of the s plane. Favorable location of the asymptotes will not alone suffice, however, since the imaginary part of R_1 must be small enough to force some segments of the primary root locus into the left-half plane. They might otherwise fail completely to cross the $j\omega$ axes. As in some previous examples, the maximum damping ratio possible may be computed by drawing a

a)



b)

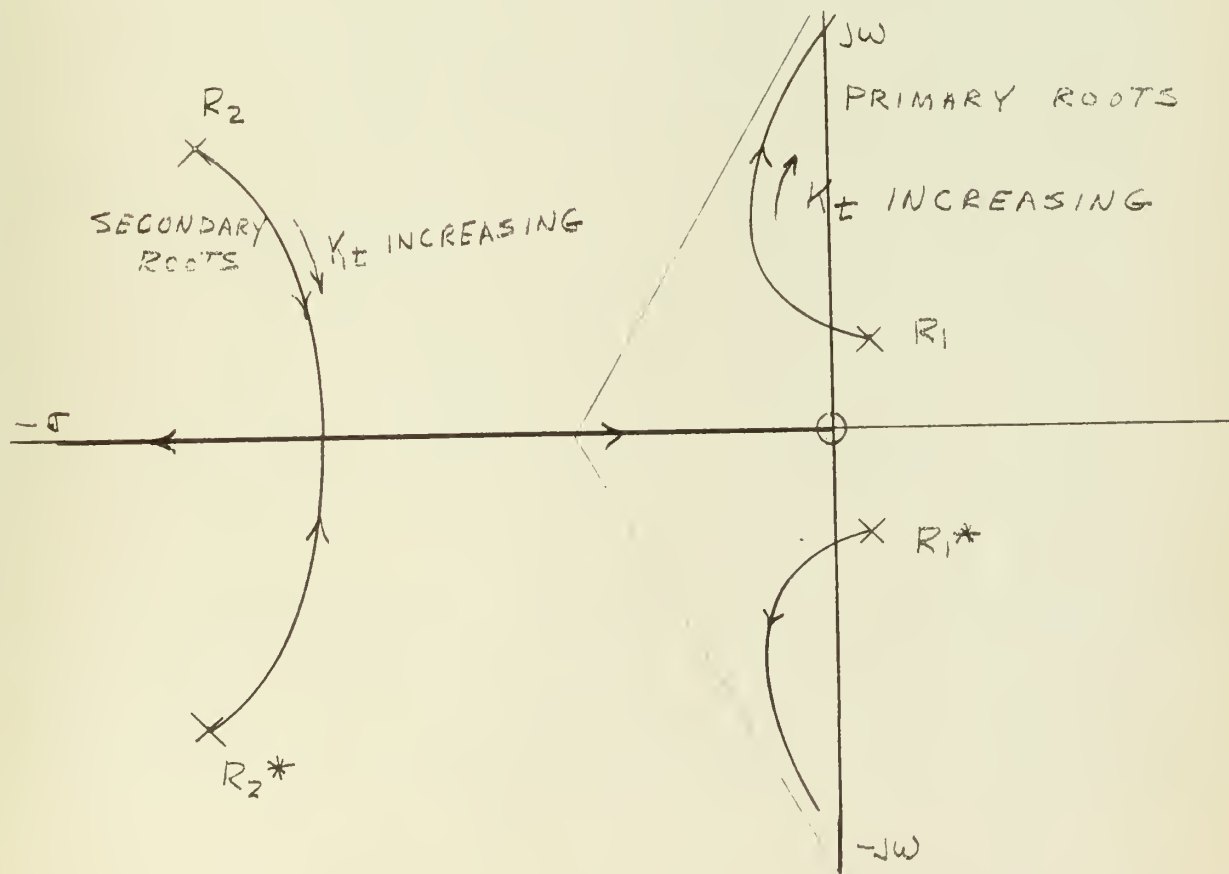


FIGURE 27 POSSIBLE ROOT LOCI FOR THE COMPENSATED SYSTEM OF EXAMPLE D (COMPENSATOR IN PATH II)

line from the origin tangent to the primary root locus segment, and by finding the cosine of the angle made by the intersection of this line with the negative real axis. The magnitudes of the transient oscillating frequency and the resonant frequency appear to be relatively large. This factor forecasts possible difficulty in meeting allowable bandwidth specifications.

The steady-state error coefficients are as listed below.

For the uncompensated system:

$$K_V = \frac{K_1 K_2}{a b c} \quad (6-23)$$

For the compensated system:

$$K_V = \frac{K_1 K_2}{a b c + K_1 K_2 K_t} \quad (6-24)$$

As found previously, this type of compensation permits retention of the type-one characteristic of the system, but reduces the steady-state accuracy of the system when the input signal is a ramp of position.

Example F

For a system where

$$G_1(s) = \frac{K_1}{(s+a)(s+b)}$$

$$G_2(s) = \frac{K_2}{s(s+c)(s+d)}$$

the uncompensated root locus is shown in Figure 28. The addition of tachometer feedback in path I results in root loci such as those shown in Figure 29. Only a few possibilities are shown. For fixed values of the

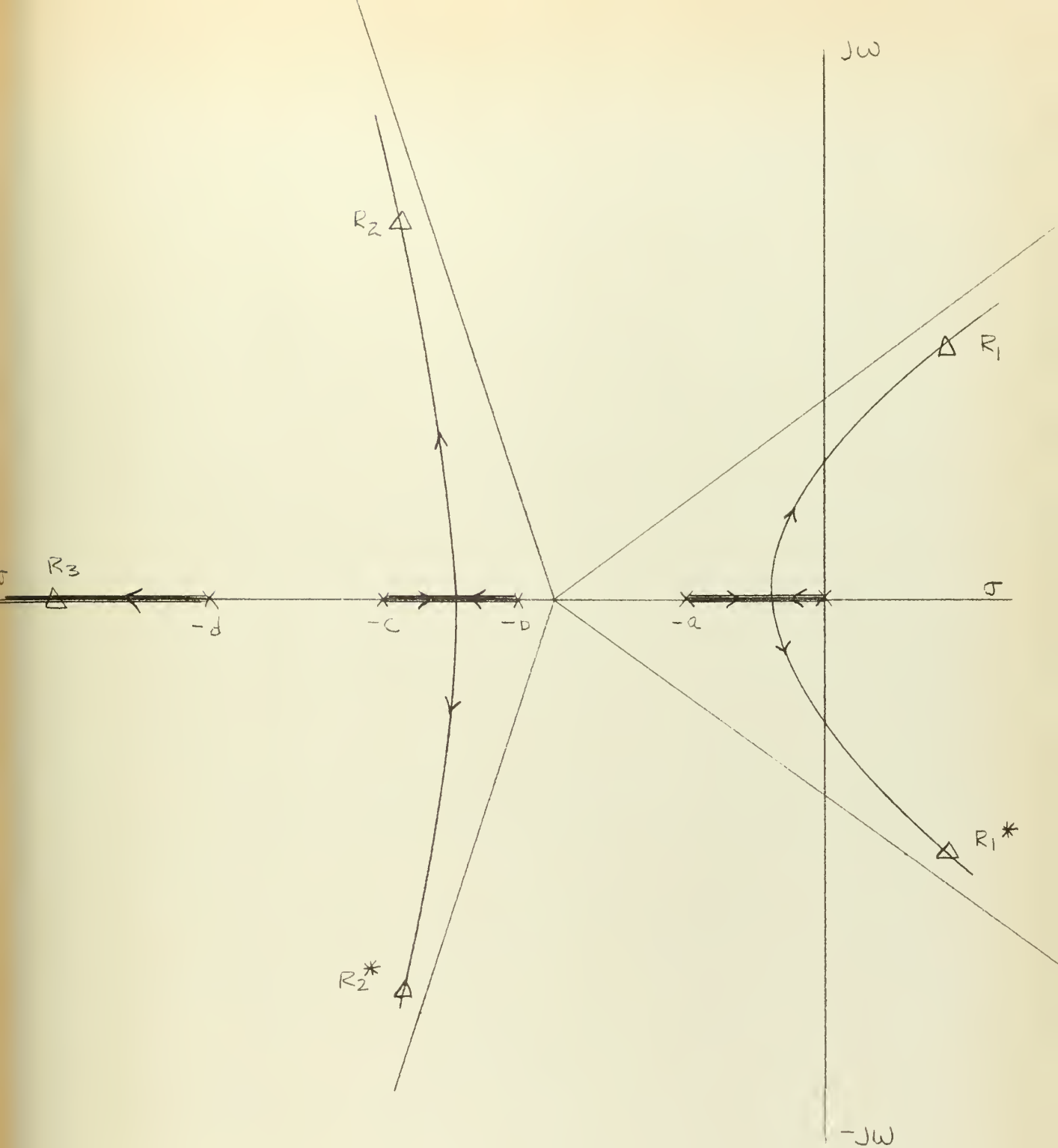


FIGURE 28 ROOT LOCUS FOR THE UNCOMPENSATED SYSTEM OF EXAMPLE F

roots of the uncompensated system, the location of the asymptote centroid will vary depending on the magnitudes of a and b . For the asymptote centroid to lie in the left half of the s plane, the condition that

$$|a| + |b| + |R_1 + R_1^*| < |R_2 + R_2^* + R_3| \quad \text{must be met.}$$

If the asymptote centroid lies in the left half of the s plane, compensation may very well be achieved by this method (see a), b) and c) of Figure 29. Consideration must be devoted to which roots dominate the transient response. It is conceivable that compensation could even be accomplished if the asymptote centroid should lie in the right half of the s plane; however, special conditions would be required for this to occur. If, for instance, the secondary root locus segments approach the right-half-plane asymptote as K_t becomes very large, it may be possible to achieve satisfactory performance by restricting the maximum allowable magnitude of K_t .

It is virtually impossible to make a general statement about a system of this complexity because there are a vast number of possible root locus configurations.

The steady-state error coefficients appear below.

For the uncompensated system:

$$K_V = \frac{K_1 K_2}{a b c d} \quad (6-25)$$

For the compensated system:

$$K_V = \frac{K_1 K_2}{a b (c d + K_2 K_t)} \quad (6-26)$$

The same system with tachometer feedback in path II has the same root locus as for the uncompensated system. A possible root locus for the com-

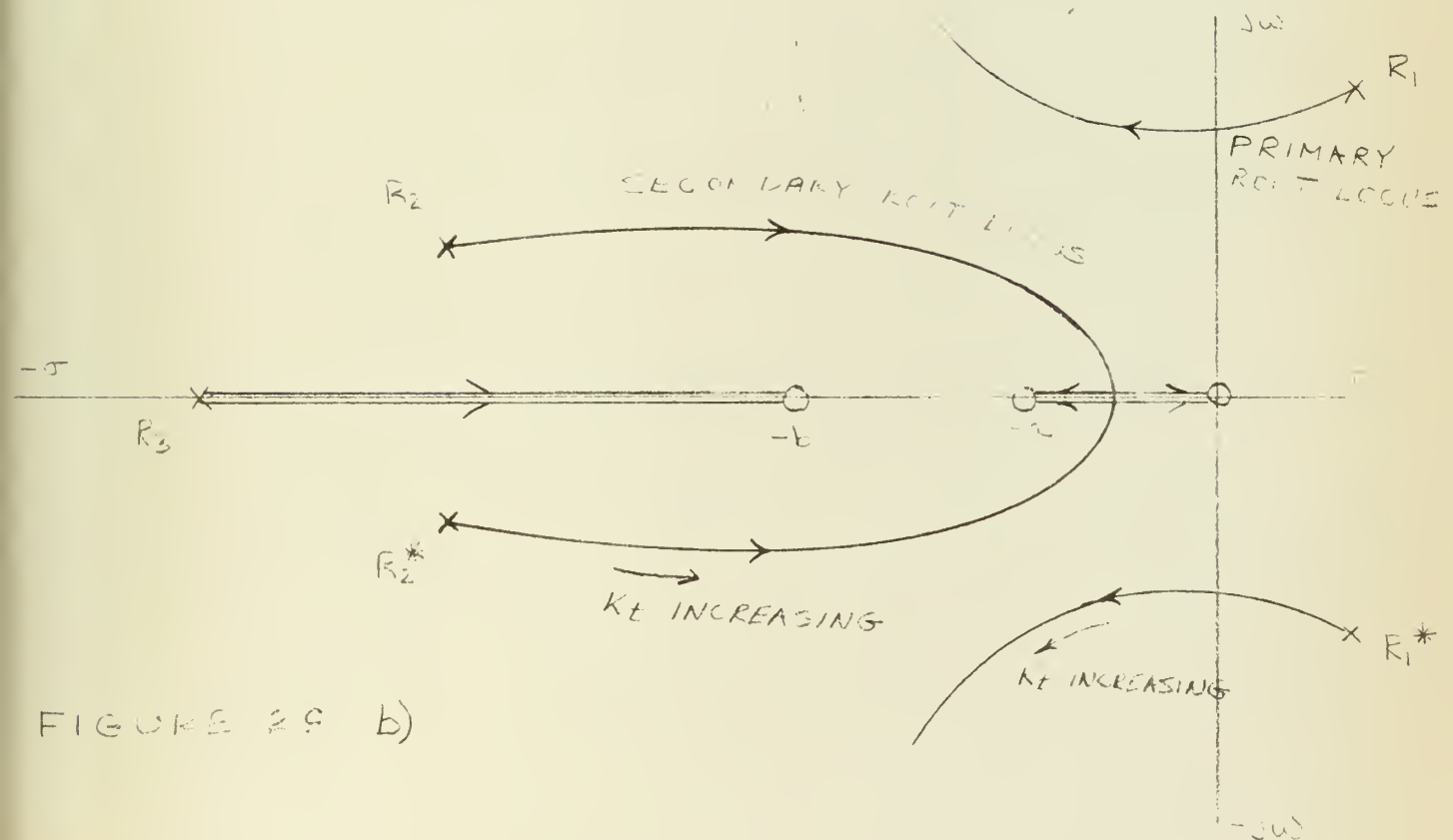
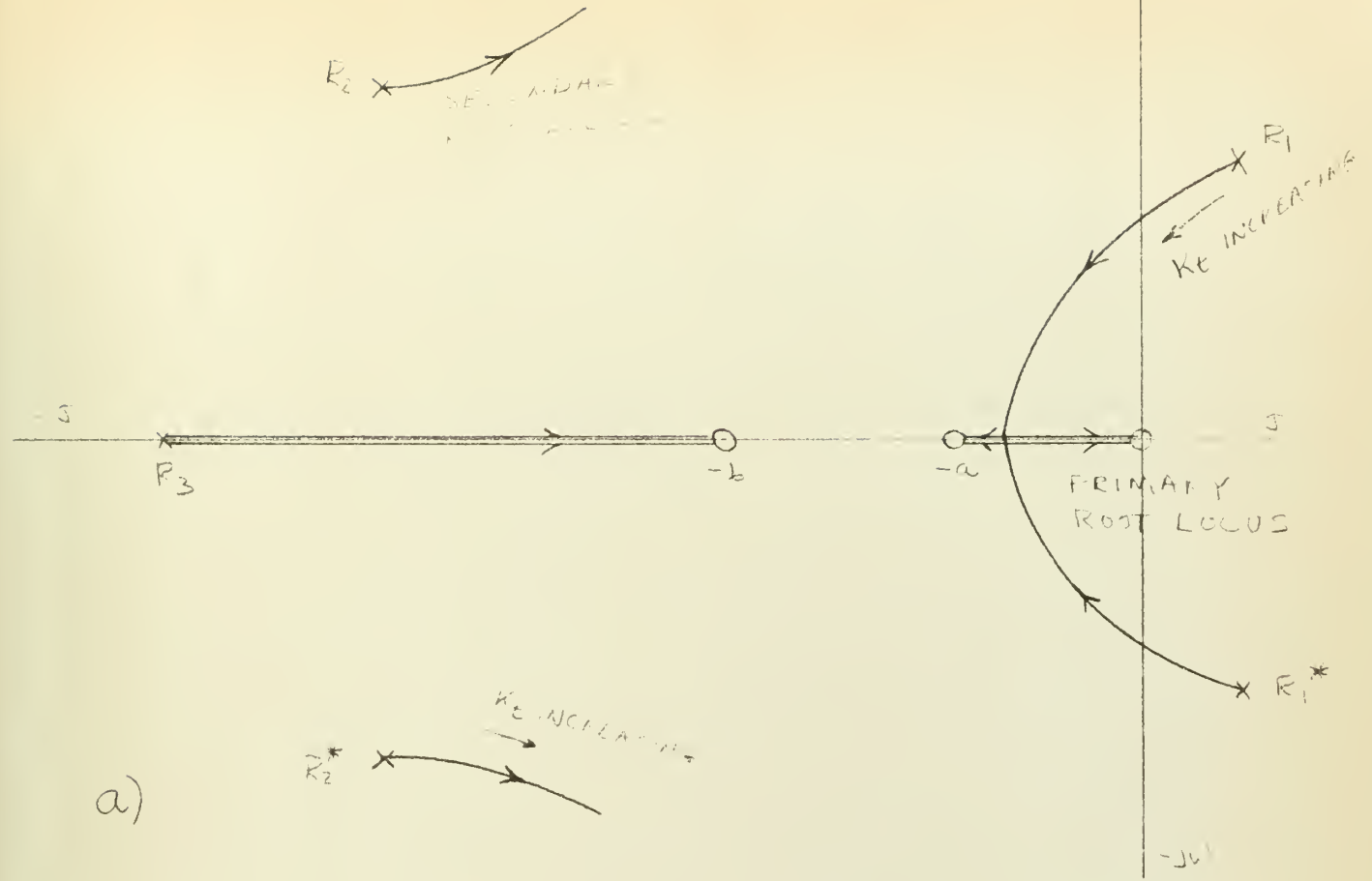


FIGURE 29 b)

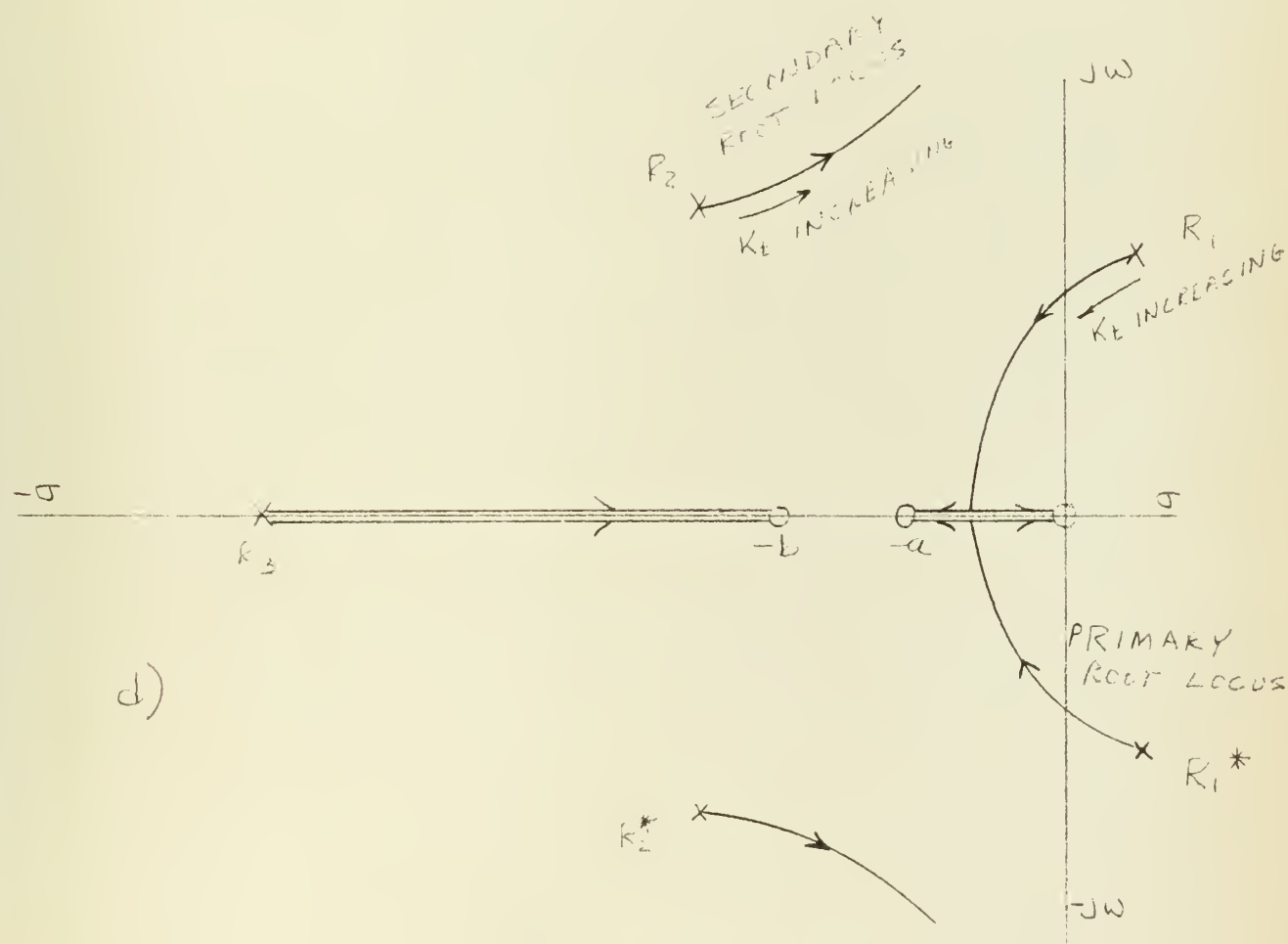
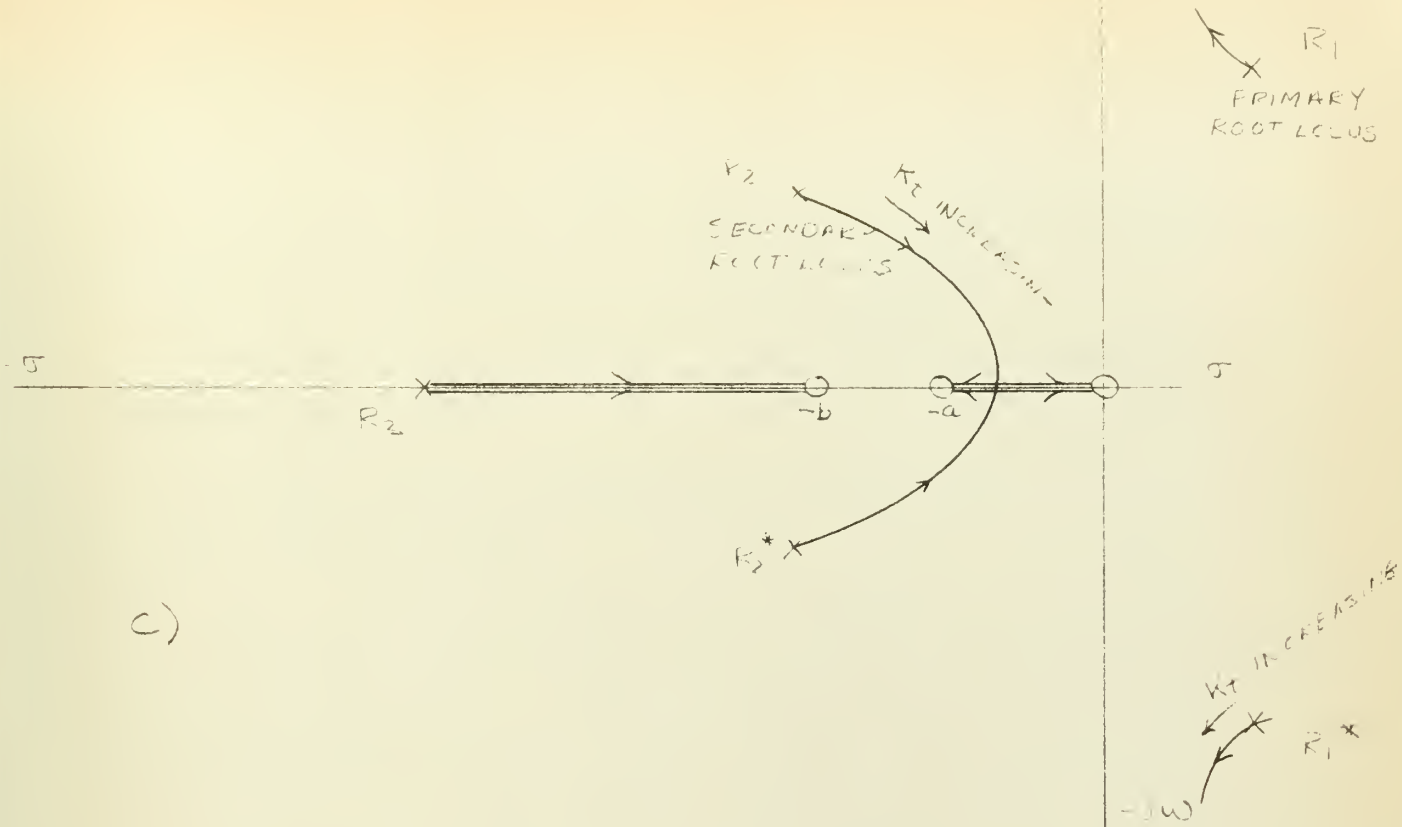


FIGURE 29 ROOT LOCI FOR THE COMPENSATED SYSTEM OF EXAMPLE F

pensated system is illustrated in Figure 30. It is observed that tachometer feedback in this path appears rather ineffective as a compensating device. It may be possible to obtain stability under certain conditions; however, even so possibilities for compensation do not appear promising.

Once more, it must be stated that it is difficult to generalize for a system of this complexity.

The steady-state error coefficients are listed below.

For the uncompensated system:

$$K_V = \frac{K_1 K_2}{a b c d} \quad (6-27)$$

For the compensated system:

$$K_V = \frac{K_1 K_2}{a b c d + K_1 K_2 K_t} \quad (6-28)$$



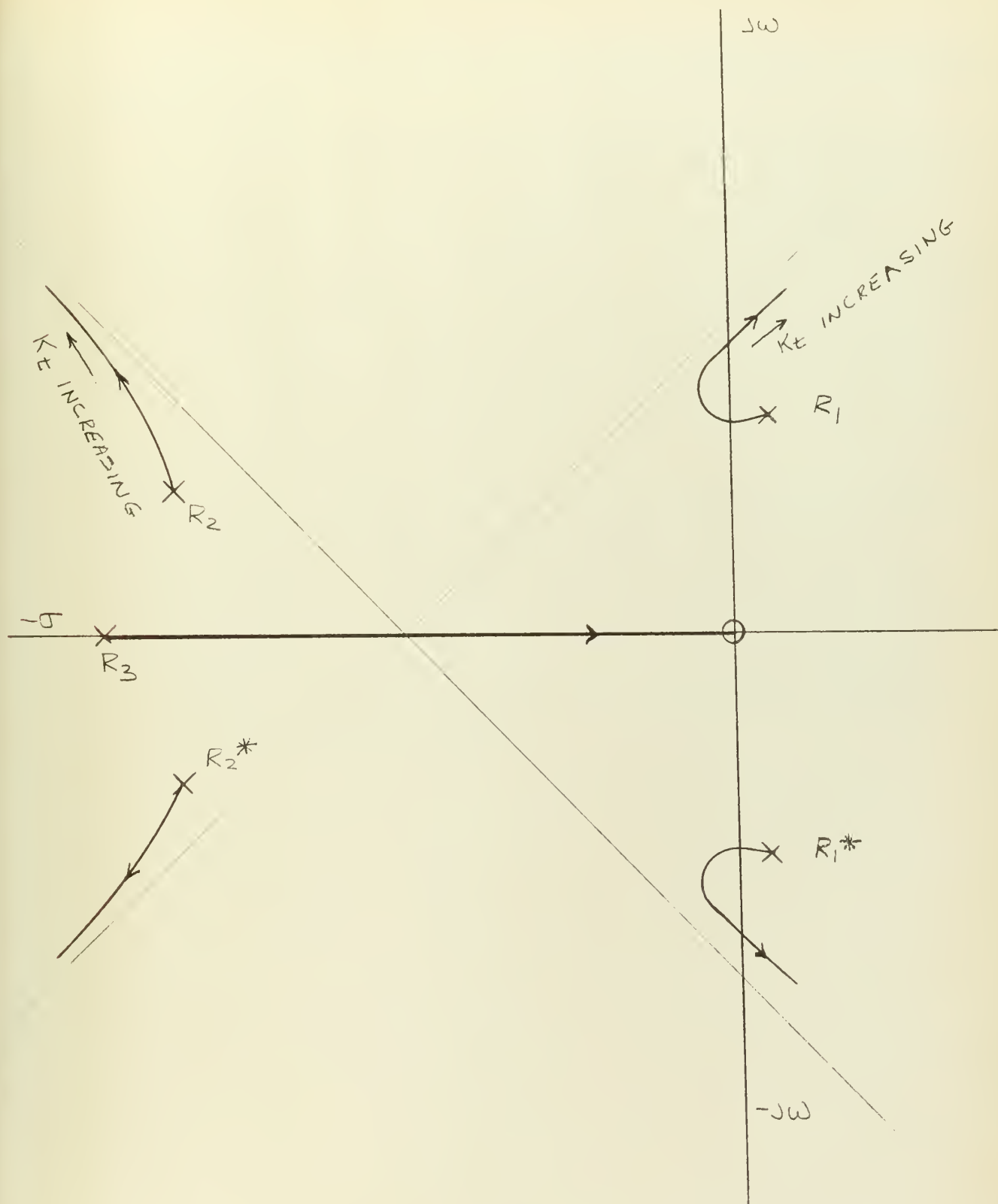


FIGURE 30 ROOT LOCUS FOR THE COMPENSATED SYSTEM OF EXAMPLE F (COMPENSATOR IN PATH II)

7. Conclusions

The method illustrated in this investigation possesses the following salient features:

1. The block-diagram manipulation enables the designer to treat the problem using standard cascade-compensation techniques.
2. The block-diagram manipulation is the same for any selected feedback path, regardless of the compensator, $G_c(s)$, used in the path. This means that once the block-diagram manipulation has been performed, the effects of using any compensator in this path may be investigated.
3. From the examples in Section 6 it is observed that as a system becomes more complex it becomes increasingly difficult to generalize concerning the feasibility of using a given compensation device; however, in many cases, this method rapidly shows the unsuitability of certain types of compensators for use in a selected compensation path.

APPENDICES

A1. Derivation of the location of the complex conjugate poles on the w plane.

From Equation (1-4c):

$$K_t S \frac{K_2}{S(S+P) + K_1 K_2}$$

$$w = f(s) = \frac{S}{\omega_N} \quad (3-1)$$

$$\frac{K_2 K_t \omega_N w}{\omega_N w (\omega_N w + P) + K_1 K_2} = -1 \quad (A1-1)$$

$$\frac{K_2 K_t \omega_N w}{\omega_N^2 w^2 + \omega_N P w + K_1 K_2} = -1 \quad (A1-1a)$$

$$\frac{\frac{K_t K_2}{\omega_N} w}{w^2 + \frac{P}{\omega_N} w + \frac{K_1 K_2}{\omega_N^2}} = -1 \quad (A1-1b)$$

$$\text{but } K_1 K_2 = \omega_N^2$$

therefore

$$\frac{\frac{K_t K_2}{\omega_N} w}{w^2 + \frac{P}{\omega_N} w + 1} = -1 \quad (A1-1c)$$

Equation (A1-1c) is of the general form;

$$\frac{K w}{(w - P') (w - P'^*)} \quad (A1-2)$$

The next step is to solve for P' and P'^* . This is done by solving for the roots of the denominator of Equation (A1-1c).

$$w = \frac{-\frac{P}{w_N} \pm \sqrt{\frac{P^2}{w_N^2} - 4}}{2} \quad (A1-3)$$

$$w = -\frac{P}{2w_N} \pm j \sqrt{1 - \frac{P^2}{4w_N^2}} \quad (A1-3a)$$

$$P' = -\frac{P}{2w_N} + j \sqrt{1 - \frac{P^2}{4w_N^2}} \quad (A1-4)$$

$$P'^* = -\frac{P}{2w_N} - j \sqrt{1 - \frac{P^2}{4w_N^2}} \quad (A1-5)$$

A2. Derivation of Equation (3-3)

From Equation (1-4c):

$$K_t = \frac{-(s^2 + ps + K_1 K_2)}{K_2 s}$$

$w = f(s) = \frac{s}{w_N} = -a + jb$ since only values in the left half of the w plane are considered here.

and, $a^2 + b^2 = 1$ since all values of w on the root locus lie on a circular arc of unit radius.

$$K_t = - \left[\frac{w_N^2 (a^2 - j2ab - b^2) + P w_N (-a + jb) + K_1 K_2}{K_2 w_N (-a + jb)} \right] \quad (A2-1)$$

$$K_t = - \left[\frac{w_N^2 (-a^3 + ja^2b - ab^2 + jb^3) + P w_N (a^2 + b^2) + K_1 K_2 (-a - jb)}{K_2 w_N (a^2 + b^2)} \right] \quad (A2-1a)$$

$$K_t = - \left[\frac{w_N^2 \{ -a(a^2 + b^2) + jb(a^2 + b^2) \} + P w_N + K_1 K_2 (-a - jb)}{K_2 w_N} \right] \quad (A2-1b)$$

$$K_t = - \left[\frac{\omega_N^2(-a+jb) + P\omega_N + K_1 K_2(-a-jb)}{K_2 \omega_N} \right] \quad (\text{A2-1c})$$

since $K_1 K_2 = \omega_N^2$

$$K_t = - \left[\frac{-2a\omega_N^2 + P\omega_N}{K_2 \omega_N} \right] \quad (\text{A2-1d})$$

$$K_t = \frac{2a\omega_N - P}{K_2} \quad (\text{A2-1e})$$

From Figure A2-1, it can be seen that:

$$\mathcal{J} = \cos \left[\cos^{-1} a \right] \quad (\text{A2-2})$$

$$\mathcal{J} = a \quad (\text{A2-3})$$

Therefore:

$$K_t = \frac{2\mathcal{J}\omega_N - P}{K_2} \quad (\text{3-3})$$



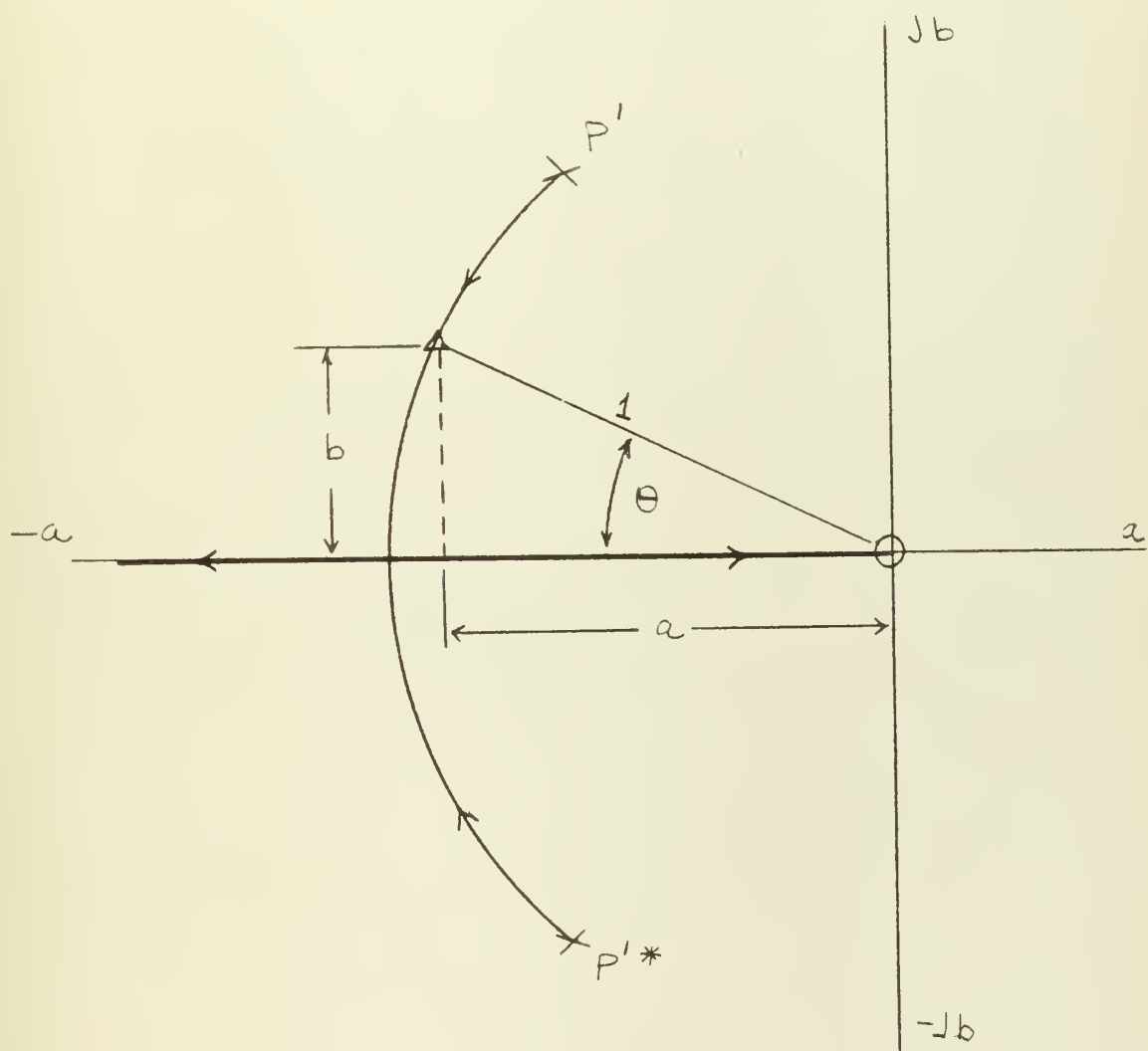


FIGURE A2-1 NORMALIZED ROOT LOCUS

A3. Derivation of Equation (3-3) by block-diagram reduction

From Equation (1-4c):

$$K_t K_2 S = -S(S+P) - K_1 K_2 \quad (\text{A3-1})$$

$$S^2 + (P + K_2 K_t) S + K_1 K_2 = 0 \quad (\text{A3-2})$$

Equation (A3-2) is of the general second-order form:

$$S^2 + 2\zeta\omega_N S + \omega_N^2 = 0 \quad (\text{3-7})$$

Therefore:

$$P + K_2 K_t = 2\zeta\omega_N \quad (\text{A3-3})$$

Solving for K_t :

$$K_t = \frac{2\zeta\omega_N - P}{K_2} \quad (\text{3-3})$$

A4. Determination of the relationship for K_v

If the block diagram of Figure 1 is reduced to the form shown in

Figure A4-1:

$$G_F(s) = \frac{G_1(s) G_2(s)}{1 + G_2(s) G_c(s)} \quad (\text{A4-1})$$

if:

$$G_1(s) = K_1$$

$$G_2(s) = \frac{K_2}{s(s+p)}$$

$$G_c(s) = K_t s$$

$$G_F(s) = \frac{\frac{K_1 K_2}{s(s+p)}}{1 + \frac{K_2 K_t s}{s(s+p)}} \quad (\text{A4-2})$$

$$G_F(s) = \frac{K_1 K_2}{s(s+p) + K_2 K_t s} \quad (\text{A4-2a})$$

$$G_F(s) = \frac{K_1 K_2}{s [s + P + K_2 K_t]} \quad (\text{A4-2b})$$

The s multiplying the polynomial in brackets in the denominator indicates this is a type-1 system. That is, there will be no steady-state output position error for a step-position input. However, there will be an output steady-state position error for a ramp-position input. The ratio of the error signal, ϵ , to the magnitude of the input signal, A , is:

$$\frac{\epsilon}{A} = \frac{1}{K_V} \quad (\text{A4-3})$$

$$\Theta_R(s) = \frac{A}{s^2} \quad (\text{A4-4})$$

where:

$$K_V = \lim_{s \rightarrow 0} s G_F(s) \quad (\text{A4-5})$$

Therefore, for this case:

$$K_V = \frac{K_1 K_2}{P + K_2 K_t} \quad 1/\text{SECOND} \quad (\text{A4-6})$$

A5. Derivation of second-order system parameters when utilizing the complex variable $w = f(s)$

For the system illustrated in Figure 1 after the transformation of variable ($w = f(s) = s/P$) has been performed, the characteristic equation is:

$$w(w + 1 + \frac{K_2 K_t}{P^2}) = 0 \quad (\text{A5-1})$$

$$w^2 + \left(\frac{P + K_2 K_t}{P} \right) w + \frac{K_1 K_2}{P^2} = 0 \quad (\text{A5-1a})$$

Equation (A5-1a) may be put in the form:

$$w^2 + 2 \zeta(w) \omega_N(w) + \omega_N^2(w) = 0 \quad (\text{A5-2})$$

where :

$$\omega_N(w) = \frac{1}{P} \sqrt{K_1 K_2} \quad (5-2)$$

$$\zeta(w) = \frac{P + K_2 K_t}{2 \sqrt{K_1 K_2}} \quad (5-3)$$

and

$$K_V(w) = \lim_{w \rightarrow 0} w \left[\frac{K_1 K_2 / P^2}{w(w + 1 + \frac{K_2 K_E}{P})} \right] \quad (5-4a)$$

$$K_V(w) = \frac{1}{P} \frac{K_1 K_2}{P + K_2 K_E} \quad (5-4)$$

A6. Derivation of the locus of $K_v(w)$ on the w plane

Referring to Figure A2-1, at point R:

$$|J(w)| = |\cos \theta| = \left| \frac{-a}{\sqrt{a^2 + b^2}} \right| \quad (\text{A6-1})$$

$$J(w) = \frac{-a}{\sqrt{a^2 + b^2}} \quad (\text{A6-1a})$$

$$w_N(w) = \sqrt{a^2 + b^2} \quad (\text{A6-2})$$

$$K_v(w) = \frac{w_N(w)}{2J(w)} \quad (5-6)$$

$$K_v(w) = \frac{a^2 + b^2}{-2a} \quad (\text{A6-3})$$

$$-2a K_v(w) = a^2 + b^2 \quad (\text{A6-4})$$

$$a^2 + b^2 + 2a K_V(w) = 0 \quad (\text{A6-5})$$

$$a^2 + 2a K_V(w) + b^2 = 0 \quad (\text{A6-5a})$$

Completing the square:

$$a^2 + 2a K_V(w) + K_V(w)^2 + b^2 = K_V(w)^2 \quad (\text{A6-6})$$

$$(a + K_V(w))^2 + b^2 = K_V(w)^2 \quad (\text{A6-7})$$

This is a circle centered at $a = -K_V$, and with a radius of K_V .

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First-derivative feedback compensation o



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